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UNIFORMLY ERGODIC PROBABILITY MEASURES

Jorge Galindo, Enrique Jordá, and Alberto Rodríguez-Arenas

Abstract: Let G be a locally compact group and μ be a probability measure on G. We consider the convolution operator $\lambda_1(\mu)\colon L_1(G) \to L_1(G)$ given by $\lambda_1(\mu)f = \mu * f$ and its restriction $\lambda_1^0(\mu)$ to the augmentation ideal $L_1^0(G)$. Say that μ is uniformly ergodic if the Cesàro means of the operator $\lambda_1^0(\mu)$ converge uniformly to 0, that is, if $\lambda_1^0(\mu)$ is a uniformly mean ergodic operator with limit 0, and that μ is uniformly completely mixing if the powers of the operator $\lambda_1^0(\mu)$ converge uniformly to 0.

We completely characterize the uniform mean ergodicity of the operator $\lambda_1(\mu)$ and the uniform convergence of its powers, and see that there is no difference between $\lambda_1(\mu)$ and $\lambda_1^0(\mu)$ in these regards. We prove in particular that μ is uniformly ergodic if and only if G is compact, μ is adapted (its support is not contained in a proper closed subgroup of G), and 1 is an isolated point of the spectrum of μ. The last of these three conditions can actually be replaced by μ being spread out (some convolution power of μ is not singular). The measure μ is uniformly completely mixing if and only if G is compact, μ is spread out, and the only unimodular value in the spectrum of μ is 1.

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