

NEWTON–OKOUNKOV BODIES AND PICARD NUMBERS ON SURFACES

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Abstract: We study the shapes of all Newton–Okounkov bodies $\Delta_v(D)$ of a given big divisor D on a surface S with respect to all rank 2 valuations v of $K(S)$.

We obtain upper bounds for, and in many cases we determine exactly, the possible numbers of vertices of the bodies $\Delta_v(D)$. The upper bounds are expressed in terms of Picard numbers and they are *birationally invariant*, as they do not depend on the model \tilde{S} where the valuation v becomes a flag valuation.

We also conjecture that the set of all Newton–Okounkov bodies of a single ample divisor D determines the Picard number of S , and prove that this is the case for Picard number 1, by an explicit characterization of surfaces of Picard number 1 in terms of Newton–Okounkov bodies.

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Key words: valuation, blowup, Newton–Okounkov body, algebraic surface, Picard number.