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ON AN ALMOST SHARP LIOUVILLE-TYPE THEOREM FOR FRACTIONAL NAVIER–STOKES EQUATIONS

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Abstract: We investigate existence, Liouville-type theorems, and regularity results for the 3D stationary and incompressible fractional Navier–Stokes equations: in this setting the usual Laplacian is replaced by its fractional power $(-\Delta)^{\frac{\alpha}{2}}$ with $0 < \alpha < 2$. By applying a fixed-point argument, weak solutions can be obtained in the Sobolev space $\dot{H}^{\frac{\alpha}{2}}(\mathbb{R}^3)$ and if we add an extra integrability condition, stated in terms of Lebesgue spaces, then we can prove for some values of α that the zero function is the unique smooth solution. The additional integrability condition is almost sharp for $3/5 < \alpha < 5/3$. Moreover, in the case $1 < \alpha < 2$ a gain of regularity is established under some conditions, although the study of regularity in the regime $0 < \alpha \leq 1$ seems for the moment to be an open problem.

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