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PORCUPINE-QUOTIENT GRAPHS, THE FOURTH PRIMARY COLOR, AND GRADED COMPOSITION SERIES OF LEAVITT PATH ALGEBRAS

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Abstract: If E is a directed graph, K is a field, and I is a graded ideal of the Leavitt path algebra $L_K(E)$, then I is completely determined by a pair (H,S) of two sets of vertices of E, called an admissible pair, and one writes $I = I(H,S)$ in this case. The ideal I is graded isomorphic to the Leavitt path algebra of the *porcupine graph* of (H,S) and the quotient $L_K(E)/I$ is graded isomorphic to the Leavitt path algebra of the *quotient graph* of (H,S). We present a construction which generalizes both the porcupine and the quotient constructions and enables one to consider quotients of graded ideals: if (H,S) and (G,T) are admissible pairs such that $(H,S) \leq (G,T)$ (in the sense which corresponds exactly to $I(H,S) \subseteq I(G,T)$), we define the *porcupine-quotient graph* $(G,T)/(H,S)$ such that its Leavitt path algebra is graded isomorphic to the quotient $I(G,T)/I(H,S)$.

Using the porcupine-quotient construction, the existence of a graded composition series of $L_K(E)$ is equivalent to the existence of a finite increasing chain of admissible pairs of E, starting with the trivial pair and ending with the improper pair, such that the quotient of two consecutive pairs is cofinal (a graph is cofinal exactly when its Leavitt path algebra is graded simple). We characterize the existence of such a chain with a set of conditions on E which also provides an algorithm for obtaining a composition series. The conditions are presented in terms of four types of vertices which are all "terminal" in a certain sense. Three of the four types are often referred to as the three primary colors of Leavitt path algebras. The fourth primary color in the title of this paper refers to the fourth type of vertices. As a corollary of our results, every unital Leavitt path algebra has a graded composition series.

We show that the existence of a composition series of E is equivalent to the existence of a suitably defined composition series of the graph monoid M_E as well as a composition series of the talented monoid M_E^Γ. We also show that an ideal of M_E^Γ is minimal exactly when it is generated by the element of M_E^Γ corresponding to a terminal vertex. We characterize graphs E such that only one or only two out of three possible types (periodic, aperiodic, or incomparable) appear among the composition factors of M_E^Γ.

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