ON THE STRONG SUBDIFFERENTIABILITY OF HOMOGENEOUS POLYNOMIALS AND (SYMMETRIC) TENSOR PRODUCTS

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Abstract: We study the (uniform) strong subdifferentiability of norms of Banach spaces $\mathcal{P}(^N X, Y^*)$ of all continuous N-homogeneous polynomials and tensor products of Banach spaces, namely $X \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X$ and $\widehat{\otimes}_{\pi_s, N} X$. Among other results, we characterize when the norms of spaces $\mathcal{P}(^N \ell_p, \ell_q)$, $\mathcal{P}(^N l_{M_1}, l_{M_2})$, and $\mathcal{P}(^N d(w, p), l_{M_2})$ are strongly subdifferentiable. Analogous results for multilinear mappings are also obtained. Since strong subdifferentiability of a dual space implies reflexivity, we improve some known results in [**38, 48, 49**] (in the spirit of Pitt's compactness theorem) on the reflexivity of spaces of N-homogeneous polynomials and N-linear mappings. Concerning the projective (symmetric) tensor norms, we provide positive results by considering the subsets U and U_s of elementary tensors on the unit spheres of $X \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X$ and $\widehat{\otimes}_{\pi_s, N} X$, respectively. Specifically, we prove that the norms of $\widehat{\otimes}_{\pi_s, N} \ell_2$ and $\ell_2 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi \ell_2$ are uniformly strongly subdifferentiable on U_s and U, and that the norms of $c_0 \widehat{\otimes}_{\pi_s} c_0$ and $c_0 \widehat{\otimes}_\pi c_0$ are strongly subdifferentiable on U_s and U in the complex case.