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TOPOLOGICAL DIMENSION ZERO AND SOME RELATED PROPERTIES

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Abstract: In this paper, we introduce and study the C*-algebras with property (IC) and with other related properties. We prove that, surprisingly, residual (IC) is equivalent to topological dimension zero (and to another property), and that in the class of C*-algebras with topological dimension zero, pure infiniteness and strong pure infiniteness coincide, providing a partial positive answer to a question of Kirchberg and Rørdam in [**12**]. We also show that these last two properties are equivalent to weak pure infiniteness and to local pure infiniteness, in the residual (IS) case, giving a particular affirmative answer to an open question of Blanchard and Kirchberg in [**2**]. We prove, in particular, that in the class of purely infinite C*-algebras, the following properties are all equivalent: residual (IC), topological dimension zero, the ideal property, the weak ideal property, residual (IF), and residual (SP). We show that crossed products by finite solvable groups preserve the class of all separable C*-algebras with topological dimension zero (resp., the weak ideal property).

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Key words: C*-algebra, residual (IC), topological dimension zero, (strongly, weakly, locally) purely infinite, the (weak) ideal property, crossed product.