Publ. Mat. **69** (2025), 473–509 DOI: 10.5565/PUBLMAT6922511

GROUND STATE SOLUTIONS OF THE SCHRÖDINGER–POISSON–SLATER EQUATION WITH DOUBLE CRITICAL EXPONENTS

Chunyu Lei and Yutian Lei

Abstract: This paper is concerned with the Schrödinger–Poisson–Slater (SPS) equation with double critical exponents. Such exponents appear in the Coulomb–Sobolev inequality, one being the Sobolev exponent and the other being called the Coulomb exponent here. We study the existence of nontrivial solutions of the SPS equation. This can be done by solving a variational problem with lack of compactness which is caused by these two critical exponents. Although the concentration compactness principle can be used to deal with the lack of compactness caused by the Sobolev exponent, it seems difficult to handle the other lack of compactness caused by the Coulomb exponent. Here we employ the Nehari–Pohozaev manifolds instead of the direct argument of concentration compactness on the Coulomb–Sobolev space to overcome this difficulty and prove that the equation possesses ground state solutions in these manifolds.

2020 Mathematics Subject Classification: 35J20, 35A23, 35Q55, 35J61.

Key words: Schrödinger–Poisson–Slater equation, ground state, Nehari–Pohozaev manifold, Coulomb–Sobolev inequality, double critical exponents.