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PICARD GROUPS OF QUASI-FROBENIUS ALGEBRAS AND A QUESTION ON FROBENIUS STRONGLY GRADED ALGEBRAS

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Dedicated to Şerban Raianu on his 70th birthday

Abstract: Our initial aim was to answer the question: does the Frobenius (symmetric) property transfer from a strongly graded algebra to its homogeneous component of trivial degree? Related to it, we investigate invertible bimodules and the Picard group of a finite-dimensional quasi-Frobenius algebra R. We compute the Picard group, the automorphism group, and the group of outer automorphisms of a 9-dimensional quasi-Frobenius algebra which is not Frobenius, constructed by Nakayama. Using these results and a semitrivial extension construction, we give an example of a symmetric strongly graded algebra whose trivial homogeneous component is not even Frobenius. We investigate associativity of isomorphisms $R^* \otimes_R R^* \simeq R$ for quasi-Frobenius algebras R, and we determine the order of the class of the invertible bimodule H^* in the Picard group of a finite-dimensional Hopf algebra H. As an application, we construct new examples of symmetric algebras.

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Key words: quasi-Frobenius algebra, Frobenius algebra, symmetric algebra, invertible bimodule, Picard group, strongly graded algebra, Hopf algebra, Nakayama automorphism.

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