



## SELF-ADJOINT TRACES ON THE PEDERSEN IDEAL OF $C^*$ -ALGEBRAS

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**Abstract:** In order to circumvent a fundamental issue when studying densely defined traces on  $C^*$ -algebras – which we refer to as the Trace Question – we initiate a systematic study of the set  $T_{\mathbb{R}}^1(A)$  of self-adjoint traces on the Pedersen ideal of  $A$ . The set  $T_{\mathbb{R}}(A)$  is a topological vector space with a vector lattice structure, which in the unital setting reflects the Choquet simplex structure of the tracial states. We establish a form of Kadison duality for  $T_{\mathbb{R}}(A)$  and compute  $T_{\mathbb{R}}(A)$  for principal twisted étale groupoid  $C^*$ -algebras. We also answer the Trace Question positively for a large class of  $C^*$ -algebras.

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**Key words:** trace space, non-unital  $C^*$ -algebra, Pedersen ideal.