



QUANTUM METRIC CHOQUET SIMPLICES

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Abstract: Precipitating a notion emerging from recent research, we formalise the study of a special class of compact quantum metric spaces. Abstractly, the additional requirement we impose on the underlying order unit spaces is the Riesz interpolation property. In practice, this means that a ‘quantum metric Choquet simplex’ arises as a unital C^* -algebra A whose trace space is equipped with a metric inducing the w^* -topology, such that tracially Lipschitz elements are dense in A . This added structure is designed for measuring distances in and around the category of stably finite classifiable C^* -algebras, and in particular for witnessing metric and statistical properties of the space of approximate unitary equivalence classes of unital embeddings of A into a stably finite classifiable C^* -algebra B . As for examples, we recall the construction of classifiable C^* -algebraic quantum metric Bauer simplices that function as noncommutative spaces of observables of compact connected metric spaces (X, ρ) . We also explain how to build non-Bauer examples by forming ‘tracial quantum crossed products’ associated with topological dynamical systems on (X, ρ) , and we use classification to show that continuous fields of quantum spaces are obtained by continuously varying either the dynamics or the metric. In the case of deformed isometric actions, we show that equivariant Gromov–Hausdorff continuity implies fibrewise continuity of the quantum structures with respect to Rieffel’s quantum Gromov–Hausdorff distance. As an example, we present a field of deformed tracial rotation algebras whose fibres are continuous with respect to a quasimetric that we call the quantum intertwining gap.

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