



## A POISONOUS EXAMPLE TO EXPLICIT RESOLUTIONS OF UNBOUNDED COMPLEXES

DOLORS HERBERA, WOLFGANG PITSCH, MANUEL SAORÍN, AND SIMONE VIRILI

**Abstract:** We show that various methods for explicitly building resolutions of unbounded complexes in fact fail when applied to a rather simple and explicit complex. We show that one way to rescue these methods is to assume Roos'  $(\text{Ab}.4^*)$ - $k$  axiom, which we adapt to encompass also resolutions in the framework of relative homological algebra. Lastly, we discuss the existence of model structures for relative homological algebra for unbounded complexes under the relative  $(\text{Ab}.4^*)$ - $k$  condition, and present a variety of examples where our results apply.

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