



DIRECT METHODS FOR A DUAL TEMPERED FRACTIONAL PARABOLIC PROBLEM

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Abstract: In this paper, we consider the dual tempered fractional parabolic problem

$$\partial_t^s u(x, t) - (\Delta + \rho)^{\frac{\alpha}{2}} u(x, t) = f(t, u(x, t)), \quad \Omega \times \mathbb{R},$$

where Ω may be \mathbb{R}^n or $\mathbb{R}_+^n := \{x \in \mathbb{R}^n \mid x_1 > 0\}$, $s \in (0, 1)$, $\alpha \in (0, 2)$, and ρ is a sufficiently small positive constant. We prove that the positive solutions are strictly increasing in the x_1 direction without assuming the solutions to be bounded. We will introduce two methods for dealing with the above dual tempered fractional parabolic problem: the method of moving planes and sliding methods. Unlike previous articles, we investigate the problems that involve both the fractional time derivative ∂_t^s and the tempered fractional Laplacian $-(\Delta + \rho)^{\frac{\alpha}{2}}$. First, by establishing the *narrow region principle* and *averaging effects* for the dual tempered fractional parabolic operators $\partial_t^s - (\Delta + \rho)^{\frac{\alpha}{2}}$, and then developing the direct moving planes to derive the monotonicity of solutions for the dual tempered fractional parabolic problem in $\mathbb{R}_+^n \times \mathbb{R}$. Second, by establishing *maximum principles in unbounded open sets* for problems involving dual tempered fractional parabolic operators, we develop direct sliding methods for the tempered fractional parabolic problem, and derive the one-dimensional symmetry of solutions to the dual tempered fractional parabolic problem in $\mathbb{R}^n \times \mathbb{R}$. As applications, we also prove the Gibbons conjecture for entire solutions to the dual tempered fractional parabolic problem.

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