- Minimal surfaces are hard to find.

Eg for trefoil  ,

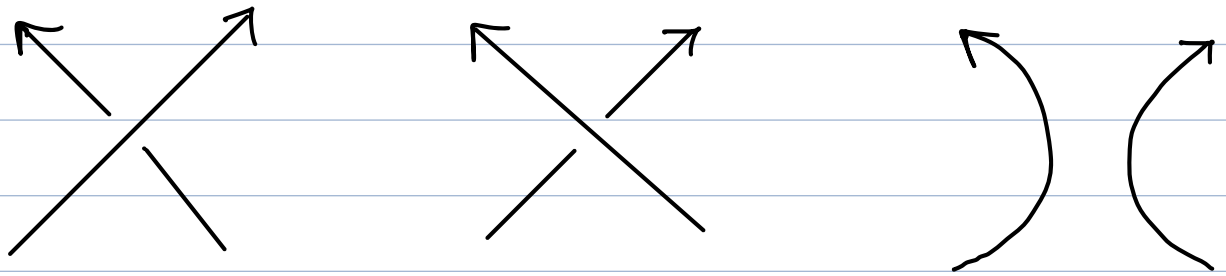
$$\text{Alexander} = 1 + z^2$$

So conjecture says every trefoil in S^3 is filled by at least one minimal surface of genus 1.

Currently an open problem to find even one such minimal surface!

Why I think the conjecture is true.

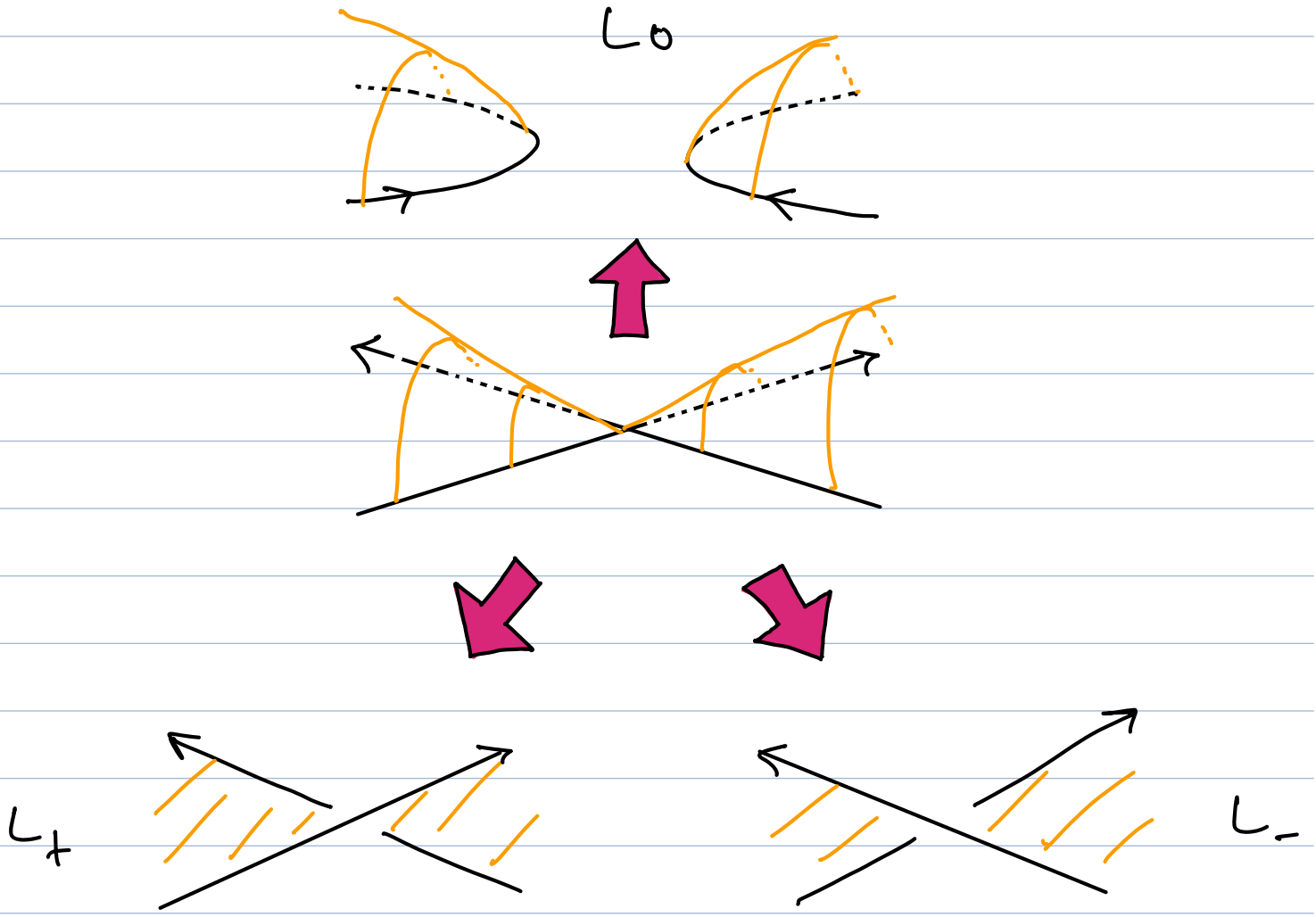
Alexander skein relation:



L_+ L_- L_0

$$\begin{cases} A(L_+) - A(L_-) = z\,A(L_0) \\ A(\text{unknot}) = 1 \end{cases}$$

Thought experiment about what happens to minimal filling as two strands of boundary are pushed through each other.



Going from L_+ to L_0 the Euler characteristic of the surface drops by 1.

Σ contributes to coefficient of $z^{1-\chi}$

So factor of z in Alexander skein relation corresponds to change of $\chi(\Sigma)$ in this picture.