

Linear combinations of iterates of Blaschke products and Peano curves

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Given an inner function f , let f^n denote the n -th iterate of f . In some sense, the functions (f^n) behave as a sequence of independent random variables on the boundary of the unit disk. This phenomenon is similar to the context of lacunary series, that is power series of the form $\sum a_k z^{n_k}$, with $n_{k+1}/n_k \geq q > 1$.

Given a sequence (a_n) of complex numbers, we are interested in the behaviour of $\sum a_n f^n$.

Recently, A Nicolau and O. Soler have obtained several results which exhibit the parallelism existing between the behaviour of lacunary series and the behaviour of linear combinations of the iterates of an inner function. For instance, a nice version of the Central Limit Theorem was obtained in [2] and some remarkable facts concerning the membership of the function $\sum a_n f^n$ to some spaces of analytic functions in terms of the coefficients is well described in [1].

As in the case of sums of independent random variables, the asymptotic behaviour of $\sum a_n f^n$ is governed by the convergence of $\sum |a_n|^2$. If last series is divergent, then $\sum a_n f^n$ has radial limit almost nowhere, but if it is finite, then $\sum a_n f^n$ belongs to BMOA, so it has nontangential limit almost everywhere and this limit coincides with $\lim_{N \rightarrow \infty} \sum_{n=1}^N a_n f^n(\zeta)$ a.e. $\zeta \in \partial\mathbb{D}$.

The purpose of this talk is to explore a little bit more the case $\sum |a_n|^2 < \infty$ following the ideas exposed in [3], when f is a finite Blaschke product.

We will show that in this case, if $\sum |a_n| = \infty$, any $w \in \mathbb{C}$ is the nontangential limit of $\sum a_n f^n$ at some point $\zeta \in \partial\mathbb{D}$.

On the other hand we will see that if $\sum |a_n| < \infty$, but the convergence is slow enough (in some sense), then the image of the unit circle under $\sum a_n f^n$ is a Peano curve.

This work is a joint collaboration with Artur Nicolau.

References

- [1] A. Nicolau. *Convergence of linear combinations of iterates of an inner function*. Preprint 2021.
- [2] A. Nicolau, O. Soler i Gibert. *A central limit theorem for inner functions*. Preprint 2020.
- [3] M. Weiss *Concerning a theorem of Paley on lacunary power series*. Acta Math. **102** (1959), 225–238.