Barcelona Analysis Seminar **2021–2022**

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Room. ONLINE

Online streaming (Microsoft Teams). Click here to join.

A free boundary problem for the heat equation/caloric measure

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In his breakthrough result, it was shown by Dahlberg that the L^2 Dirichlet problem for the Laplacian (harmonic functions) is solvable in the region above a Lipschitz graph. Dahlberg did this by showing a local reverse Hölder inequality for the Poisson kernel in such domains. It was conjectured by Hunt that the same should be true in the parabolic setting, that is, a 'parabolic Lipschitz' graph domain should be sufficient for L^2 (or L^p) solvability of the heat equation; however, Kaufman and Wu provided a counter-example. Later, sufficient additional conditions for (some) L^p and L^2 solvability in the parabolic graph setting were introduced by Lewis and Murray and Lewis and Hofmann, respectively.

Together with Hofmann, Martell and Nyström, we have recently shown that the condition introduced by Lewis and Murray is necessary for the L^p solvability of the Dirichlet problem (for some p). More specifically, the (additional) necessary and sufficient condition is that the graph function have a half-order time derivative in BMO. The heart of our proof combines two ideas: (1) Fefferman-Stein characterization of BMO by Littlewood-Paley extensions and (2) using the ideas from Alt and Caffarelli to show that the Green function has 'nice' level sets in a large region. In the proof, the graph of the level sets (roughly) play the role of the Littlewood-Paley extension.

The talk will be purely at the level of 'ideas', as a series of technical lectures given by Hofmann is available (on YouTube).