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# Optimal regularity for the fully nonlinear thin obstacle problem

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The thin obstacle problem for an elliptic operator $L$ with zero obstacle is

$$\begin{cases} Lu = 0 & \text{in } B_1 \setminus \{x_n = 0, u = 0\} \\ Lu \leq 0 & \text{in } B_1 \\ u \geq 0 & \text{on } B_1 \cap \{x_n = 0\}. \end{cases} \tag{1}$$

When $L$ is the Laplacian, this is also called the Signorini problem, and it is now a classical free boundary problem. In this case, the $C^{1,\alpha}$ (one-sided) regularity of solutions was first proved by Caffarelli in 1979, and it was not until 2004 that Athanasopoulos and Caffarelli were able to prove the optimal regularity of solutions, $C^{1,1/2}$. This was done by observing that solutions to the thin obstacle problem satisfy certain monotonicity formulas, available only because $L = \Delta$.

In this talk we deal with the nonlinear generalization consisting in taking $Lu = F(D^2u)$ for some convex uniformly elliptic fully nonlinear operator $F$.

Under such conditions, the $C^{1,\alpha}$ regularity was proved in 2016, and then a study of regular points was performed in 2017 by Ros-Oton and Serra, where they are locally a $C^1$ manifold. Nonetheless, due to the lack of monotonicity formulas, the optimal regularity of solutions was still not known.

In this talk, we present our recent results together with M. Colombo and X. Ros-Oton, where we show what the optimal regularity of (1) is. In particular, we prove that, if in addition $F$ is rotationally invariant, then solutions are always $C^{1,1/2}$.