Barcelona Analysis Seminar **2021–2022**

URL *(provisional)*. https://sites.google.com/view/seminari-analisi-barcelona/2021-2022

Date. January 31, 2022

Time. 15:00 CET

Room. Room B1, Universitat de Barcelona

Online streaming (Zoom). https://ub-edu.zoom.us/j/95538016558

Linearity of homogeneous solutions to degenerate elliptic equations in dimension three

Pablo Mira
Universidad Politécnica de Cartagena

In this talk we will give an answer to the following problem: which linear elliptic equations $\sum a_{ij}u_{ij} = 0$ in $\mathbb{R}^3$ satisfy that all their degree-one homogeneous solutions are linear functions? Alexandrov conjectured in 1956 that this linearity result should hold for every such elliptic equation. However, Martinez-Maure found in 2001 a striking counterexample to this claim. On the other hand, Han, Nadirashvili and Yuan proved in 2003 that the Alexandrov conjecture holds if the equation is uniformly elliptic. In this talk, we will extend this theorem to the degenerate elliptic case. Instead of uniform ellipticity, we will just assume the following much weaker condition, which is actually sharp by Martinez-Maure's example. Let $\mathcal{O}$ be a connected open subset of $\mathbb{S}^2 \subset \mathbb{R}^3$ with the property that it intersects any configuration of four disjoint geodesic semicircles of $\mathbb{S}^2$; for example, $\mathcal{O}$ can be chosen as a thin neighborhood of a geodesic in $\mathbb{S}^2$. Assume that, on $\mathcal{O}$, the ratio between the largest and smallest eigenvalues of the coefficient matrix (a_{ij}) of the PDE lies in L^1_{loc}. Then, we prove that any degree-one homogeneous solution to the equation is linear. This is a joint work with Jose A. Galvez, from Granada.