Quantitative differentiability on uniformly rectifiable sets

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A basic fact of Lipschitz functions is that they are differentiable almost everywhere. This is Rademacher's theorem. It says a lot about the asymptotic behaviour of Lipschitz functions at small scales (where they look affine) but not much at any definite scale. How long do we need to wait for the the smoothness of f to kick in and make it look like an affine map? Dorronsoro's theorem is a quantification of Rademacher's which tells us: not long. Indeed, a Lipschitz function looks approximately affine at most scales (in some precise sense). Dorronsoro's theorem has plenty of applications. It is, for example, a cornerstone of the theory of uniform rectifiability. In this talk, I will discuss how to extend it to functions defined on non-necessarily-smooth subsets of Euclidean space (i.e. uniform rectifiable sets). If time permits, I will present some applications, in particular some Sobolev-type trace theorems for domains with rough boundaries. Based on a joint work with Jonas Azzam and Mihalis Mourgoglou.