Barcelona Analysis Seminar 2021–2022

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Room. CRM A1 (Universitat Autònoma de Barcelona)

Online streaming (Zoom). Click here to join.

Gabor orthonormal bases, tiling and periodicity

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Given a Gabor orthonormal basis of $L^2(\mathbb{R})$

$$\mathcal{G}(g,T,S) := \left\{ g(x-t)e^{2\pi i s x} : g \in L^2(\mathbb{R}),\, t \in T,\, s \in S \right\},$$

we study periodicity properties of the translation and modulation sets T and S. In particular, we show that if the window function g is compactly supported, then T and S must be periodic sets, i.e., of the form

$$T = a\mathbb{Z} + \{t_1, \dots, t_n\}, \qquad S = b\mathbb{Z} + \{s_1, \dots, s_m\}.$$

To achieve this, we first obtain a result of independent interest: if the system $\mathcal{G}(g,T,S)$ is an orthonormal basis of $L^2(\mathbb{R})$, then both $|g|^2$ and $|\widehat{g}|^2$ tile $\mathbb{R}$ by translations (when translated along the sets T and S, respectively), and moreover,

$$\sum_{t\in T} |g(x-t)|^2 = D(T), \qquad \sum_{s\in S} |\widehat{g}(x-s)|^2 = D(S), \qquad \text{a.e. } x \in \mathbb{R},$$

where $D(\Lambda)$ denotes the uniform density of a set $\Lambda \subset \mathbb{R}$.

Partial results towards the Liu-Wang conjecture are also obtained. Based on a joint work with Nir Lev (Bar-Ilan University, Israel).