Barcelona Analysis Seminar **2022–2023**

URL. `https://mat.uab.cat/web/seminarianalisi/`

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Online streaming. Click here to join.

L^p-solvability of the Poisson problem and its applications to the regularity problem

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We introduce the concept of L^p-solvability of the Poisson problem

$$\begin{cases} -\operatorname{div} A\nabla w = H - \operatorname{div} F, & \text{in } \Omega, \\ w = 0, & \text{on } \partial\Omega, \end{cases}$$

under certain natural quantitative assumptions on H and F, with its corresponding new estimate (new even for the Laplacian), and study several applications. By L^p solvability, we mean uniform bounds on the L^p norm of the non-tangential maximal function of w. An analogous concept is classical and central for the theory of boundary value problems for *homogeneous* second-order elliptic PDEs. Our main application is towards the L^p Dirichlet-regularity problem for elliptic operators $-\operatorname{div} A\nabla$ whose matrix A satisfies the Dahlberg-Kenig-Pipher condition (this is, roughly speaking, a Carleson measure condition on $|\nabla A|^2 \operatorname{dist}(\cdot, \partial\Omega)$), in the geometric generality of bounded Corkscrew domains with uniformly rectifiable boundaries. This solves an open problem from 2001. Other applications include new characterizations of the L^p-solvability of the Dirichlet problem, and a non-tangential maximal function estimate for the gradient of the Green's function, in Corkscrew domains with Ahlfors-regular boundaries. This is joint work with Mihalis Mourgoglou and Xavier Tolsa.