

John conditions, Harnack chains and boundary Poincaré inequalities

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We consider connections between the local John condition, the Harnack chain condition and weak boundary Poincaré inequalities in an open set $\Omega \subset \mathbb{R}^{n+1}$ with n -dimensional Ahlfors–David regular boundary. First, we show that if Ω satisfies both the local John condition and the exterior corkscrew condition, then Ω also satisfies the Harnack chain condition (and hence, is a chord-arc domain). Second, we show that if Ω is a 2-sided chord-arc domain, then the boundary $\partial\Omega$ supports a Heinonen–Koskela-type weak p -Poincaré inequality for any $1 \leq p < \infty$. We also discuss the optimality of our assumptions and some follow-up questions. This is a joint work with Xavier Tolsa.