Barcelona Analysis Seminar **2022–2023**

URL. `https://mat.uab.cat/web/seminarianalisi/`

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Room. B3 (Universitat de Barcelona)

Online streaming (Zoom). Click here to join.

Cascade phenomena for a complete integrable system

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The cubic Szegő equation has been introduced as a toy model for totally non dispersive evolution equations. It is the following evolution equation

$$i\partial_t u(x,t) = \Pi(|u|^2 u)(x,t),\ u(x,0) = u_0(x),\ x \in \mathbb{T},\ t \in \mathbb{R}.$$

Here Π is the orthogonal projection from $L^2(\mathbb{T})$ on the Hardy space $H^2(\mathbb{T})$ or in other words the operator

$$\Pi : \begin{cases} L^2(\mathbb{T}) & \to \quad H^2(\mathbb{T}) \\ u := \sum_{k\in\mathbb{Z}} \hat{u}(k)\mathrm{e}^{ikx} & \mapsto \quad \sum_{k\geq 0} \hat{u}(k)\mathrm{e}^{ikx} \end{cases}$$

It turns out that it is a complete integrable Hamiltonian system for which we built a non linear Fourier transform giving an explicit expression of the solution. This non linear Fourier transform is constructed through an inverse spectral theorem for Hankel operators which is interesting by itself.

This explicit formula allows to study the dynamics of the solutions. Among other things, we will explain how we prove cascade phenomena for a dense set of smooth initial data in large Sobolev spaces.

This is a survey from joint works with Patrick Gérard, université Paris-Saclay.

Keywords: Hamiltonian system, integrability, Hankel operators.