

Good behaviour of the harmonic measure on boundaries of small dimension and a distance function equation

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How does the behaviour of the harmonic measure on the boundary $\partial\Omega$ of the domain Ω characterise the geometry of the boundary? For the classical setting, when the boundary $\partial\Omega$ is $n - 1$ -dimensional, after decades of gradual progress, the question was settled after the final effort due to J. Azzam, S. Hoffman, M. Mourgoglou, J. M. Martell, S. Mayboroda, X. Tolsa and A. Volberg (2015-2016). However, when the boundary has a smaller dimension, to characterise its geometry similarly, even the notion of the harmonic measure itself had to be reintroduced. It was done recently by G. David, S. Mayboroda, J. Feneuil, and other coauthors with the aid of degenerate elliptic operators, the nicest of which is

$$L_\alpha = -\operatorname{div} D_\alpha^{-n+d+1} \nabla,$$

where D_α is a regularised distance function. We will discuss the key obstacle to completing the characterisation of the nice geometry of the boundary for this case. It turns out to be a question, which one can state easily, concerning solutions of the equation $L_\alpha D_\alpha = 0$. We will also discuss some recent progress concerning its resolution.