Barcelona Analysis Seminar **2022–2023**

URL. `https://mat.uab.cat/web/seminarianalisi/`

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Room. CRM A1 (Universitat Autònoma de Barcelona)

Online streaming. Click here to join.

New progress on solvability of Regularity problem for elliptic operators with coefficients satisfying large Carleson condition

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Recall that when $Lu = \operatorname{div}(A\nabla u)$ and coefficients A satisfy Carleson condition (that is $|\nabla A|^2\delta$ is a Carleson measure) then the corresponding elliptic measure is A_∞ and hence the L^p Dirichlet problem is solvable for some large $p < \infty$. There is also an analogous smallness result, namely that on C^1 domains L^p Dirichlet problem is solvable for all $1 < p < \infty$, provided $|\nabla A|^2\delta$ is a vanishing Carleson measure.

In the case of Regularity problem, the analogous smallness result was established in Dindoš-Pipher-Rule. We have recently (jointly with S. Hofmann and J. Pipher) established an n-dimensional reduction that relates solvability of Regularity problem to the solvability of Regularity problem for a block-form operator.

This reduction has allowed us to fully resolve the question of solvability of Regularity problem with coefficients satisfying large Carleson condition in all dimensions and also the Neumann problem in dimension 2 in the interval for $1 < p < 1 + \epsilon$.

Remarkably, Mourgoglou, Poggi and Tolsa considered the same question from a different perspective using new ideas that allow extrapolation of the small Carleson result of Dindoš-Pipher-Rule using improved version of Mourgoglou Tolsa decomposition of domains to Lipschitz subdomains (developed for originally for the Regularity problem for Laplacian). This method allows to consider the Regularity problem for more general domains (beyond Lipschitz).

I compare the two methods and outline possible further open questions that might be addressed thanks to these exciting developments.