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Regularity for the prescribed Lorentzian mean curvature equation, and the Born-Infeld model

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In electrostatic Born-Infeld theory, the potential u_ρ generated by a charge distribution ρ on $\mathbb{R}^m$ (typically, a measure) is required to minimize the action

$$\int_{\mathbb{R}^m} \left(1 - \sqrt{1 - |D\psi|^2}\right) \mathrm{d}x - \langle \rho, \psi \rangle$$

among functions with $|D\psi| \leq 1$ and a suitable decay at infinity. Formally, the Euler-Lagrange equation $(\mathcal{BI})$ prescribes ρ as the Lorentzian mean curvature of the graph of u_ρ in Minkowski spacetime $\mathbb{L}^{m+1}$, making contact with General Relativity. For instance, if ρ is a finite sum of Dirac deltas, then the graph of u_ρ is a maximal hypersurface with singularities in $\mathbb{L}^{m+1}$. While the existence/uniqueness of u_ρ follows from standard variational arguments, the singularity of the Lagrangian density when $|D\psi| = 1$ makes the solvability of $(\mathcal{BI})$ an open problem for most classes of measures ρ. In this talk, I will report on a recent joint work with J. Byeon, N. Ikoma and A. Malchiodi, where we study the solvability of $(\mathcal{BI})$ and the regularity of u_ρ under mild conditions on ρ. One of the main problems is the possible presence of light rays in the graph of u_ρ, which will be discussed in detail.