

On the number of lower sets with fixed cardinality

Kristina Oganesyan

Universitat Autònoma de Barcelona

We call a set $S \subset \mathbb{Z}_{+}^{d}$, $d \geq 2$, a lower set if for any $\mathbf{x} = (x_{1}, ..., x_{d}) \in \mathbb{Z}_{+}^{d}$ the condition $\mathbf{x} \in S$ implies $\mathbf{x}' = (x'_{1}, ..., x'_{d}) \in S$ for all $\mathbf{x}' \in \mathbb{Z}_{+}^{d}$ with $x'_{i} \leq x_{i}$, $1 \leq i \leq d$. One can also think of a *d*-dimensional lower set as of a union of unit cubes $\prod_{i=1}^{d} [k_{i}, k_{i} + 1]$, $k_{i} \in \mathbb{Z}_{+}$, such that in each direction any cube leans either on another one or on the coordinate hyperplane. A simple example of a three-dimensional lower set can be seen in the figure below.



Lower sets are crucial objects in approximation theory, harmonic analysis, and in various problems of physics. We will discuss upper and lower bounds for the number $p_d(n)$ of d-dimensional lower sets of cardinality n. The talk does not require any prior knowledge of the topic.