Barcelona Analysis Seminar **2022–2023**

URL. https://mat.uab.cat/web/seminarianalisi/

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Online streaming (Zoom). Click here to join.

Regularity of stable solutions to semilinear elliptic problems

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A classical problem in PDE concerns the regularity of stable solutions to nonlinear elliptic equations. This turns to be a delicate question, even for the apparently simple semilinear equation $-\Delta u = f(u)$ with f smooth. For this class of solutions, regularity depends on the dimension n of the space. When $n \geq 10$, examples of singular stable solutions have been known for a long time. By contrast, when $n \leq 9$, it was recently shown in [Cabré, Figalli, Ros-Oton, and Serra, Acta Math. 224 (2020)] that *all* stable solutions must be bounded (and hence smooth).

In this talk, we consider more general equations $-Lu = f(u)$, where L is a uniformly elliptic operator with variable coefficients. Our main theorem establishes the boundedness of stable solutions in $C^{1,1}$ domains for the optimal range $n \leq 9$. This result is new even for the Laplacian, for which a C^3 regularity assumption on the domain was needed.