

Hardy–Littlewood fractional maximal operators on homogeneous trees

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Let X be a homogeneous tree equipped with the standard graph distance and the counting measure, and denote by \mathcal{M}^γ , $\gamma \in (0, 1]$, the Hardy-Littlewood fractional maximal operator on this space. We discuss for which couples (p, q) the operator \mathcal{M}^γ maps continuously $L^p(X)$ to $L^q(X)$, and for which it does not. To obtain the positive results, we prove some endpoint estimates extending the weak $(1, 1)$ boundedness of \mathcal{M}^1 (proved by Cowling, Meda, and Setti and, independently, by Naor and Tao), and the restricted weak type $(2, 2)$ boundedness of $\mathcal{M}^{1/2}$ (proved by Veca) to values of γ different from 1 and $1/2$. We will discuss the optimality of the results and highlight which problems remain open and comment on them. The talk is based on a joint work with Federico Santagati.