Multiple solutions to the nonlocal Liouville equation in $\mathbb{R}$

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We construct multiple solutions to the Liouville type equation

$$(-\Delta)^{\frac{1}{2}}u = k(x)e^u, \quad \text{in } \mathbb{R}.$$

More precisely, for k of the form $k(x) = 1 + \epsilon\kappa(x)$ with $\epsilon \in (0,1)$ small and $\kappa \in C^{1,\alpha}(\mathbb{R}) \cap L^\infty(\mathbb{R})$ for some $\alpha > 0$, we prove the existence of multiple solutions to the above equation bifurcating from the so-called Aubin-Talenti bubbles. These solutions provide examples of flat metrics in the half-plane with prescribed geodesic curvature $k(x)$ on its boundary. Moreover, they imply the existence of multiple ground state soliton solutions for the Calogero-Moser derivative NLS. The talk is based on joint works with L. Battaglia (Roma), M. Cozzi (Milano) and A. Pistoia (Roma).