Barcelona Analysis Seminar **2023–2024**

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A numerical method for the solution of boundary value problems on convex planar domains

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The Unified Transform Method (UTM) was pioneered in the early '90s by A. S. Fokas and I. M. Gel'fand in their study of the numerical solution of boundary value problems for elliptic PDEs and for a large class of nonlinear PDEs. The UTM provides a connection between the Fourier Transform method for linear PDEs (FT) and its nonlinear counterpart, namely the Inverse Spectral method – also known as Non Linear Fourier Transform method (NLFT). At the heart of the matter is a new derivation of the FT method for linear equations in one and two (space) variables that follows the same conceptual steps needed to implement the NLFT method for a class of nonlinear evolution equations, thus pointing to a unified approach to the numerical solution of linear and nonlinear PDEs.

From the very beginning, the UTM has attracted a great deal of interest in the applied mathematics community. A multitude of versions of the original method have since been developed, each dealing with a specific family of equations. Here we focus on a 2003 result of A.S. Fokas and A.A. Kapaev pertaining to the study of boundary value problems for the Laplacian on convex polygons: their original approach relied on a variety of tools (spectral analysis of a parameter-dependent ODE; Riemann-Hilbert techniques, etc.) but it was later observed by D. Crowdy that the method can be recast within a complex function-theoretic framework which, in turn, expands the applicability to so-called circular domains (domains bounded by arcs of circles, with line segments being a special case).

We extend the original approach of Fokas and Kapaev for polygons, to arbitrary convex domains. It turns out that ellipses (which are not circular in the sense of Crowdy) are of particular relevance in applications to engineering because the most popular heat exchangers (namely the shell-and-tube exchangers) have elliptical cross section.

In this talk I will describe a complex function-theory based new algorithm for convex domains, and will highlight the numerical challenges that arise when implementing it.

This is joint work with J. Hulse (Syracuse University), S. Llewellyn Smith (UCSD & Scripps Institute of Oceanography) and Elena Luca (The Cyprus Institute).

More information at: `https://mat.uab.cat/web/seminarianalisi/`