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Restriction of Békollé-Bonami Weights

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A positive weight w on $\mathbb{D}$ is a B_p weight if

$$\sup_{I \subseteq \mathbb{T}} \left(\frac{1}{|I|^2} \int_{S(I)} w \, dA \right) \left(\frac{1}{|I|^2} \int_{S(I)} w^{-\frac{1}{p-1}} \, dA \right)^{p-1} < \infty,$$

where, for any interval $I \subset \mathbb{T}$, $S(I) := \{z \in \mathbb{D} \,|\, z/|z| \in I, 1 - |z| \leq |I|\}$ denotes the *Carleson box* based at I, and $|I|$ is the length of I. This definition dates back to 1978, where Bekolle and Bonami proved in [2] that those are precisely the weights that characterize the boundedness of the Bergman projection on $L^p(\mathbb{D}, wdA)$. Namely, B_p weights play the same role that Muckenhoupt A_p weights play for the Hilbert transform on $L^p(\mathbb{T}, d\theta)$. However, the analogy stops at a certain point, since B_p weights can be much more irregular than Muckenhoupt weights. In particular, they can easily fail to satisfy the reverse Hölder and self-improvement properties that A_p-weights enjoy. Recently, it was discovered that these desirable properties can be recovered [1] if we consider weights $w \in B_p$ which are of *bounded hyperbolic oscillation*, that is, weights for which there is a constant $L > 0$ such that

$$\big| \log w(z) - \log w(\zeta) \big| \leq L\big(1 + \beta(z, \zeta)\big), \quad \text{a.e. } z, \zeta \in \mathbb{D}, \tag{0.1}$$

where β denotes the hyperbolic metric on $\mathbb{D}$. Békollé–Bonami weights satisfying (0.1), as well as their connection with the Bloch space, were further explored in [4].

In this talk we characterize the restriction of B_p weights of bounded hyperbolic oscillation on subsets of the unit disc. This is done by solving a related problem for dyadic B_p weights, and by then averaging properly the dyadic solutions. Our main result is not true for general B_p wieghts, while an analogous result for A_p weights is a well known theorem of Wolff. As a by-product of our techniques we also prove that any $w \in B_p$ of bounded hyperbolic oscillation, $p > 1$, can be factored as $w = w_1 w_2^{1-p}$, where our novel contribution is to ensure that the weights $w_1, w_2 \in B_1$ are also of bounded hyperbolic oscillation. This type of factorization problem within a subclass of weights has previously been raised by Borichev [3, Remark 2], where he considered a different class of well-behaved B_p weights.

Time permitting, we will also cover a related restriction problem, by considering the values that a Bloch function can attain on an interpolating sequence.

This is a joint work with Adrian LLinares and Karl-Mikael Perfekt.

References

[1] Alexandru Aleman, Sandra Pott, and María Carmen Reguera, *Characterizations of a limiting class B_∞ of Békollé-Bonami weights*, Rev. Mat. Iberoam. **35** (2019), no. 6, 1677–1692.

[2] David Békollé and Aline Bonami, *Inégalités à poids pour le noyau de Bergman*, C. R. Acad. Sci. Paris Sér. A-B **286** (1978), no. 18, A775–A778.

[3] Alexander Borichev, *On the Bekollé-Bonami condition*, Math. Ann. **328** (2004), no. 3, 389–398.

[4] Adem Limani and Artur Nicolau, *Bloch functions and Bekollé-Bonami weights*, Indiana Univ. Math. J. **72** (2023), no. 2, 381–407.

More information at: `https://mat.uab.cat/web/seminarianalisi/`