Barcelona Analysis Seminar **2024–2025**

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Room: UB iA (Universitat de Barcelona)

Sharp Invertibility in Quotient Algebras of H^∞

Pascal J. Thomas

Institut de Mathématiques de Toulouse

Given an inner function $\Theta \in H^\infty(\mathbb{D})$ and $[g]$ in the quotient algebra $H^\infty/\Theta H^\infty$, its quotient norm is $\|[g]\| := \inf\{\|g + \Theta h\|_\infty, h \in H^\infty\}$. Let g be normalized so that $\|[g]\| = 1$. Given $\varepsilon > 0$, we would like to be able to find $\delta > 0$ so that if $|g(\lambda)| \geq 1 - \delta$ for any λ such that $\Theta(\lambda) = 0$, then $\|[g]^{-1}\| < 1 + \varepsilon$.

This happens if and only if:

$$\lim_{t\to 1} \inf\left\{|\Theta(z)| : z \in \mathbb{D}, \rho(z, \Theta^{-1}\{0\}) \geq t\right\} = 1,$$

where ρ is the usual pseudohyperbolic distance in the disc, $\rho(z, w) := \left|\frac{z-w}{1-z\bar{w}}\right|$.

Call this Sharp Invertibility Property (SIP). We prove that an inner function Θ is SIP if and only if for any $\varepsilon > 0$, the set $\{z : 0 < |\Theta(z)| < 1 - \varepsilon\}$ cannot contain hyperbolic disks of arbitrarily large radius.

A divisor of a SIP inner function isn't always SIP; we study the functions which can be divisors of SIP inner functions, prove that any divisor of Θ is SIP if and only if any Frostman shift of Θ (itself included) is a Carleson-Newman Blaschke product, and give a Carleson-like characterization of that last property.

Joint work with Alexander Borichev, Artur Nicolau, Myriam Ounaïès.

More information at: `https://mat.uab.cat/web/seminarianalisi/`