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On zero and uniqueness sets for classical Fock spaces

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For $\alpha > 0$ and $0 \leq p < \infty$, let \mathcal{F}_α^p be the classical Fock space of entire functions, i.e.,

$$\mathcal{F}_\alpha^p := \left\{ f : \mathbb{C} \rightarrow \mathbb{C} \text{ entire} : \|f\|_{p,\alpha}^p := \int_{\mathbb{C}} |f(z)e^{-\frac{\alpha}{2}|z|^2}|^p dm(z) < \infty \right\}.$$

In [6], K. Zhu has explored various properties of zero sets in the Fock spaces \mathcal{F}_α^p . In particular, he proved that the spaces \mathcal{F}_α^p and \mathcal{F}_α^q always have different zero sets when $\beta \neq \alpha$, regardless of the values of p and q . Subsequently, he asked whether this remains true if $\alpha = \beta$ and $q \neq p$, a problem we resolved in [1], which was also independently addressed by Russell and Zhai in [5]. As far as I know, a complete characterization of this problem is still open, and even the uniqueness problem remains much more difficult!

This talk will focus on the stability problem (perturbation) of two practical examples of uniqueness sets. Additionally, we will discuss a characterization of zero sets for which every subset is also a zero set in the same Fock space \mathcal{F}_α^p ($1 \leq p < \infty$), which can be seen as a divisibility problem, as obtained in [2].

If time permits, I will also touch on new ideas involving probabilistic tools and results related to these problems [3, 4].

References

- [1] Aadi, Driss, Brahim Bouya, and Youssef Omari. "On zero sets in Fock spaces." *Journal of Mathematical Analysis and Applications* 466.2 (2018): 1299-1307.
- [2] Aadi, D., and Y. Omari. "Zero and uniqueness sets for Fock spaces." *Canadian Mathematical Bulletin* 66.2 (2023): 532-543.
- [3] Fang, Xiang, and Pham Trong Tien. "A sufficient condition for random zero sets of Fock spaces." *Archiv der Mathematik* 117 (2021): 291-304.
- [4] Kononova, Anna. "Random zero sets for Fock type spaces." *Analysis and Mathematical Physics* 13.1 (2023): 9.
- [5] Lyons, Russell, and Alex Zhai. "Zero sets for spaces of analytic functions." *Annales de l'Institut Fourier*. Vol. 68. No. 6. 2018.
- [6] Zhu, K. "Analysis on Fock spaces." *Graduate Text in Mathematics/Springer* (2012).