Barcelona Analysis Seminar **2025–2026**

Date: Thursday April 16, 2026
Time: 15:00 CET
Room: UB iA (Universitat de Barcelona)

Time-frequency localization in the Fourier Symmetric Sobolev space

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We define the Fourier symmetric Sobolev space $\mathcal{H}$ to be the space consisting of functions $f \in L^2(\mathbb{R})$ such that

$$\|f\|_{\mathcal{H}}^2 = \int_{\mathbb{R}} |f(x)|^2(1+x^2)dx + \int_{\mathbb{R}} |\hat{f}(\xi)|^2(1+\xi^2)d\xi < \infty.$$

It is a reproducing kernel Hilbert space, and we will begin the talk by deriving the formula for the reproducing kernel $K_t(x)$. We base it on the discovery that the Bargmann transform is a unitary operator from $\mathcal{H}$ to a weighted Fock space. Then we will use it to study the concentration problem in $\mathcal{H}$, namely that of maximizing the ratio

$$\frac{\int_I (1+x^2)|f(x)|^2dx + \int_J (1+\xi^2)|\hat{f}(\xi)|^2d\xi}{\int_{\mathbb{R}} (1+x^2)|f(x)|^2dx + \int_{\mathbb{R}} (1+\xi^2)|\hat{f}(\xi)|^2d\xi},$$

where $I, J \subset \mathbb{R}$ are measurable. We approach it by studying the eigenvalues of the corresponding concentration operator $T_{I,J} : \mathcal{H} \to \mathcal{H}$ defined as

$$T_{I,J}f(t) = \int_I f(x)\overline{K_t(x)}(1+x^2)dx + \int_J \hat{f}(x)\overline{\hat{K}_t(x)}(1+x^2)dx.$$

By calculating the trace of $T_{I,J}$ and the sum of the squares of its eigenvalues, we discover an unexpected phenomenon about the decay of eigenvalues of the concentration operator, i.e. the plunge region is of the same order of magnitude as the region where the eigenvalues are close to 1, contrasting the classical case of Paley–Wiener space.

Finally, in the case $I = J = (-R, R)$ we will use the weighted Paley–Wiener space of functions with $\text{supp}(\hat{f}) \subset (-R, R)$ and $\|f\|^2 = \int_{\mathbb{R}} |f(x)|^2(1+x^2)dx$, to obtain a lower bound of $4R^2$ for the number of eigenvalues of T which are close to 1.

More information at: `https://mat.uab.cat/web/seminarianalisi/`