Clifford algebras, spinors, and applications

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Horário: 2^a, 3^a, 4^a e 5^a das 10h às 12h.

Spinors play a fundamental role in Geometry and Physics. The Clifford algebra is the natural linear algebraic setting where spinors and the Spin group are formulated. This course will offer a small but solid introduction to Clifford algebras and spinors, including many examples, and a taste of some of its application within Physics and Mathematics.

By the end of the course, the students should have a hands-on knowledge of Clifford algebras and spinors, should have grasped the basics of the theory, and should be aware of some applications within Physics and Geometry.

Clifford algebras

- Preliminaires: vector spaces, algebras and their representations.
- Motivation and definition of Clifford algebra. First examples.
- Involutions of the Clifford algebra. Simplicity.
- Classification of real and complex Clifford algebras.

Spinors

- Definition of spinor and examples.
- The Clifford, Pin and Spin groups.
- Pairings in spinors. Pure spinors.
- Spin(3) and Spin(4). Spin(8) and triality.

Applications. Some of these topics will be chosen:

- Spinors and the spin in Physics.
- Pauli/Dirac matrices. Majorana/Weyl/Dirac spinors.
- The Dirac operator.
- Spin geometry.
- Dirac and generalized complex structures.

References

[1] J. Figueroa-O'Farrill. *Spin Geometry*. Lecture notes, available at http://empg.maths.ed.ac.uk/Activities/Spin, 2010.

[2] T. Friedrich. *Dirac operators in Riemannian geometry*, volume 25 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2000.

[3] D. J. H. Garling. *Clifford algebras: an introduction*, volume 78 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 2011.

[4] F. R. Harvey. *Spinors and calibrations*, volume 9 of *Perspectives in Mathematics*. Academic Press, Inc., Boston, MA, 1990.

[5] H. B. Lawson, Jr. and M.-L. Michelsohn. *Spin geometry*, volume 38 of *Princeton Mathematical Series*. Princeton University Press, Princeton, NJ, 1989. Available online at www.indiana.edu/∼ jfdavis/teaching/m721/resources/spingeometry.pdf.

[6] P. Lounesto. *Clifford algebras and spinors*, volume 286 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, second edition, 2001.

[7] E. Meinrenken. *Clifford algebras and Lie theory*, volume 58 of *A Series of Modern Surveys in Mathematics*. Springer, Heidelberg, 2013. Preliminary online version available at http://isites.harvard.edu/fs/docs/icb.topic1048774.files/clif_mein.pdf.