Clifford algebras, spinors and applications Problem Sheet 0

17 January 2016

This problem sheet consists of a sample problem, but you are expected to write your solution or ideas and submit them, so we can discuss it in class.

(In case you are meant to get a grade at the end of the course, this problem sheet will not count towards it.)

Problem 1. The following statements are not necessarily true. If you think they are true, just prove them. If you think they are false or inaccurate, say why, find a similar statement which you think is true, and try to prove it.

a)

$$\operatorname{Cl}(V \oplus V', Q \oplus Q') = \operatorname{Cl}(V, Q) \otimes \operatorname{Cl}(V', Q'),$$

where $(V \oplus V', Q \oplus Q')$, the direct sum of the quadratic vector spaces (V, Q), (V', Q'), is again a quadratic vector space.

b)

$$\mathbb{H} \otimes_{\mathbb{R}} \mathbb{H} \cong M_4(\mathbb{R}),$$

where \mathbb{H} denotes the algebra of quaternions, and $M_4(\mathbb{R})$ the algebra of 4×4 real matrices.