# Clifford algebras, spinors and applications
# Problem Sheet 1

24 January 2017

**Problem 1.** We have seen that $Cl(V \oplus V', Q \oplus Q')$ is not necessarily isomorphic to $Cl(V, Q) \otimes Cl(V', Q')$, but to the graded tensor product, $Cl(V, Q) \otimes_g Cl(V', Q')$. There are several cases, though, where we have an identification of $Cl(V \oplus V', Q \oplus Q')$ with a usual tensor product of Clifford algebras, sometimes reversing the signature. We are explaining them this week.

All of them can actually be summed up in a simple statement like this:

$$Cl(V \oplus V', Q \oplus Q') \cong Cl(V, \lambda Q) \otimes Cl(V', Q'),$$

for some scalar $\lambda$. However, some hypotheses may be needed for this to be true. It is your job to find reasonable hypotheses (not too restrictive) and prove the result. Do this for real quadratic spaces. If you feel adventurous, think about what happens in general.

**Problem 2.** A finite-dimensional representation of an algebra $A$ is a finite-dimensional vector space $V$ together with an algebra homomorphism

$$\rho : A \to End(V).$$

A representation $\rho$ is called irreducible if there is no non-trivial proper $\rho$-invariant subspace. Moreover, if $A$ is unital, we ask $\rho$ to be unital.

In this problem you have to prove that the only irreducible (unital) representation of the algebra of matrices $M_n(\mathbb{R})$ is, up to isomorphism, the standard representation $\mathbb{R}^n$. But this is a very well-known result that you could find in many places. And you should! Your job is to find several proofs, read them, and write your own personal proof, one you feel totally comfortable about. Of course, you can state some auxiliary results, as Schur's lemma.

What happens for $M_n(\mathbb{C})$ and $M_n(\mathbb{H})$?