Generalized Geometry, an introduction
Assignment 9

Weizmann Institute
Second Semester 2017-2018

There is no formal submission of the assignments but you are expected to work on them.

Problem 1. We have defined the Dorfman bracket by

$$[X+\alpha, Y+\beta] = [X,Y] + L_X\beta - i_Y d\alpha.$$

We want to find some of its symmetries. Consider a diffeomorphism $f : M \to M$, that is, a smooth map with smooth inverse. We use the notation $f_* : T \to T$ for the differential and $f_* = (f^*)^{-1}$. Consider the orthogonal bundle map

$$f_* := \begin{pmatrix} f_* & 0 \\ 0 & f_* \end{pmatrix} : T + T^* \to T + T^*.$$

- Recall the properties of push-forwards and pull-backs and prove that

$$[f_* u, f_* v] = f_*[u,v].$$

For $B \in \Omega^2(M)$, define $e^B := \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix} : T + T^* \to T + T^*$.

- When do we have $[e^B u, e^B v] = e^B[u,v]$?

Problem 2. Let $M = \mathbb{R}^3$ with coordinates (x,y,z) and consider the coordinate vector fields $\{\partial_x, \partial_y, \partial_z\}$, which generate T at every point. Consider the 1-forms $\{dx, dy, dz\}$, which are dual to the coordinate vector fields and generate T^* at every point. Define the subbundle

$$L := span(\partial_y + zdx, \partial_x - zdy, dz) \subset T + T^*.$$

- Prove that L is a maximally isotropic subbundle.
- Is L involutive with respect to the Dorfman bracket?
- Describe $L \cap T$.

Problem 3. Define a generalized complex structure as a complex Dirac structure $L \subset (T+T^*)_{\mathbb{C}}$ such that $L \cap \overline{L} = \{0\}$ and $\Gamma(L)$ is involutive with respect to the Dorfman bracket.

• Prove that the subbundle $L_J = T^{0,1} + (T^{1,0})^*$, attached to a complex structure J, is a generalized complex structure.