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Open problems about reflections in triangulated categories

Whether or not every full subcategory closed under triangles, retracts and products in a triangulated category is associated with a localization is a traditional question, appearing in earlier work on stable homotopy theory and, more recently, in the setting of derived categories. We will recall conditions under which the answer to this question is affirmative, and display a counterexample showing that it can also be negative in some cases.

As we will show, certain large-cardinal principles guarantee that the answer to the above question is affirmative in the homotopy category of every locally presentable stable model category. However, we do not know if there is any counterexample to this statement under set-theoretical assumptions incompatible with the existence of large cardinals.

A number of other problems remain open in this context. We will discuss the following ones concerning the homotopy category of spectra: Is there a set or a proper class of cohomological Bousfield classes? Is every exact localization associated with a homology theory?

A good reference for terminology and motivation is the monograph entitled *Axiomatic Stable Homotopy Theory*, by Hovey–Palmieri–Strickland, Mem. Amer. Math. Soc. 128, no. 610, 1997. Our results are included in articles in preparation with J. J. Gutiérrez, A. Neeman, and J. Rosický.

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