Main Talks

IV Seminar on Categories and Applications

Bellaterra, 6 to 9 of June of 2007

Joachim Kock, Universitat Autònoma de Barcelona

Algebraic geometry over symmetric monoidal categories (after Toën and Vaquié)

Abstract: I outline the approach of Toën and Vaquié to algebraic geometry relative to a symmetric monoidal category C. Let Aff_C denote the opposite of the category of commutative monoids in C, and define a notion of Zariski topology by taking as open immersions the flat monos of finite presentation. Now a scheme relative to C is a sheaf on Aff_C that admits a Zariski open cover by representables. The construction is functorial in C, and base change is an important aspect of the theory. When C is the category of abelian groups, the usual notion of scheme results. The interesting new cases lie below $\operatorname{Spec}\mathbb{Z}$, escaping the realm of commutative rings. When C is the category of sets we get a version of the mythical "schemes over the field with one element" ($\mathbb{F}_1$-schemes), cf. Soulé, which can be base changed to $\mathbb{Z}$ to get usual schemes. Examples of $\mathbb{F}_1$-schemes are toric varieties. There is a homotopical version of the theory, relative to a C with a Quillen model structure. This leads to different forms of homotopical algebraic geometry, including "brave new schemes". (Everything is from Toën-Vaquié, math.AG/0509684.)

Contact address: `kock@mat.uab.cat`