IMPROVING THE SKATING SYSTEM - I

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Charles Lutwidge Dodgson,
alias Lewis Carroll, 1876:

"I am quite prepared to be told, with regard to the cases I have here proposed, as I have already been told with regard to others, 'Oh, *that* is an extreme case: it could never really happen!' Now I have observed that this answer is always given instantly, with perfect confidence, and without any examination of the details of the proposed case."

The Skating system is the standard scrutineering procedure of dancesport competitions. It has been used without change since 1956 [**1**]. Lately, however, some criticisms have been pointed out. In the present paper, we discuss the existing criticisms and we propose some ways of solving them. The rest of this paper assumes that the reader has a certain familiarity with the traditional Skating system, for which we refer to [**1**] or to the more recent expositions [**2, 3, 4, 5**].

1. The main problem and two fundamental principles.

For every dance of a dancesport final, each judge ranks the couples from better to worse according to his own preferences. The problem we are dealing with is to suitably combine these multiple rankings into a final global one.

The traditional Skating system (TSS) does that in two parts. The first part combines the different judges within each dance, while the second part combines the different dances.

A central and very appreciated feature of the Skating system is its concern with the two following fundamental principles:

- Principle 1. Judges should be combined according to the opinion of an absolute majority of them.
- Principle 2. All dances should have the same weight in the final result.

In the traditional Skating system, Principle 1 is embodied in Rules 5 and 8, while Principle 2 is the matter of Rule 9 (which combines dances by simply adding the dance results). Together with these rules, the traditional Skating system includes also other rules

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URL: `http://puffinet.com/escrutini/iss1en.pdf`.

The authors are indebted to Mr. Harry Smith-Hampshire for his interesting remarks.

that have nothing to do with the fundamental principles above, but are used to break ties when the preceding rules are not enough to that effect. This is the case of Rules 6 and 7 when combining judges within each dance, and of the more famous Rules 10 and 11 when combining dances.

2. Earlier criticisms.

Criticisms against the present Skating system are as old as the system itself. At the meeting of the *Official Board of Ballroom Dancing* (now *British Dance Council*) where the Skating system was last amended, doubts were expressed about it by Frank Borrows B Sc, the only qualified mathematician present at the meeting, but his warnings were unheeded.

2.1. Arbitrary untying rules.

A very common criticism is that Rules 10 and 11 have nothing to do with Principle 2, namely that all dances should have the same weight. Because of that, some dancesport regulations give up untying the couples that obtain the same total over dances, or alternatively they do it by means of an additional round between the tied couples. Such a procedure still has a place in the regulations of the *Deutcher Tanzsportverband (DTV)*, from Germany, where it receives the name of *Majoritätssystem*.

As we shall see in § 6, Rules 6 and 7 can also be considered a possible source of undesirable effects.

2.2. Same performances and same preferences of the judges, but different results.

At first sight, the dance addition rule (Rule 9) seems reasonable enough. However, in some situations it can lead to really undesirable phenomena. Let us consider, for instance, the situation that took place in certain top international championships of the early 1950's, where most judges would agree that the British couple was the best all-round one, but their Viennese Waltz was of a lower standard. In such a situation, a six couple final would easily result in a dance summary like the one shown next at the left-hand side. In contrast, a four couple final would give a dance summary like the one shown at the right-hand side. Here S denotes the sum over dances, and P (for place or position) denotes the ordinal number after ranking the couples according to the preceding parameter.

Example A.
TSS Dance Summary.

Nº	*Dances* W	T	V	F	Q	S	P
11	1	1	6	1	1	**10**	*2*
12	2	2	1	2	2	**9**	*1*

Example A′.
TSS Dance Summary.

Nº	*Dances* W	T	V	F	Q	S	P
11	1	1	4	1	1	**8**	*1*
12	2	2	1	2	2	**9**	*2*

This is certainly most undesirable: even assuming the same performances, and the same preferences of the judges, the winner depends on how many other couples are present! By the way, in 1953 and 1954 the European Standard Professional Championships recalled only four couples into the final round; in both occasions the winners were Henry Kingston & Joy Tolhurst, from Britain, and in 1954 the runners-up were Paul & Margit Krebs, from Germany, especially praised for their Viennese Waltz.

The preceding paradox throws serious doubts about the suitability of the addition method in the present circumstances. The main fault lies in *adding ordinal numbers*. In fact, the numbers that we are adding here are simply the positions obtained in the different dances. Of course, these numbers can be used to compare couples, but they are certainly not meant to quantify their differences, as we are implying when we add them up. In other words, a 6th position does not necessarily mean a six times worse performance than a 1st position, nor the difference from 1st to 6th is necessarily five times larger than the difference from 1st to 2nd.

In principle, this problem can be solved in two different ways. A first possibility is to *replace addition by a method more suitable to ordinal numbers*. Possibly, the most suitable one is the method of paired comparisons described in [**8**]. In the preceding example, that method would amount to say that couple 11 is the winner both in B and in B′ because, when compared with any other couple, the number of dances where couple 11 beats the other couple is always larger than the number of dances where it is beaten.

A second possibility is to *replace ordinal numbers by a more quantitative measure of the degree of merit achieved in each dance*. In fact, this will be the approach used in one of the procedures proposed below.

2.3. Anomalous results because of information being discarded.

Sometimes, the results of the traditional Skating system simply do not agree with a common-sense-guided first look. Let us consider for example the following case:

Example B

	Cha-cha-cha					*Samba*					*Rumba*					*Paso Doble*					*Jive*				
Nº	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
21	1	1	1	1	1	1	1	1	1	1	1	1	3	3	2	1	1	3	3	2	1	1	3	3	2
22	6	6	3	3	3	3	3	3	6	6	2	2	2	2	6	2	2	2	2	6	2	2	2	2	6
23	4	4	2	5	4	5	5	5	3	3	5	4	5	4	4	5	5	5	5	5	5	5	6	4	5
24	3	3	6	6	5	6	6	6	2	2	6	5	6	6	1	6	6	6	4	1	6	4	5	5	1
25	5	2	4	2	6	2	2	4	4	4	3	3	1	1	3	3	3	1	1	3	3	3	1	1	3
26	2	5	5	4	2	4	4	2	5	5	4	6	4	5	5	4	4	4	6	4	4	6	4	6	4

Anybody who is familiar with the Skating system will have no trouble to check that it leads to the following dance summary.

Example B.
TSS Dance Summary.

	Dances						
Nº	C	S	R	P	J	*S*	*P*
21	1	1	2	2	2	**8**	*2*
22	2	2	1	1	1	**7**	*1*
23	3	5	4	5	6	**23**	*5*
24	6	6	6	6	5	**29**	*6*
25	5	3	3	3	3	**17**	*3*
26	4	4	5	4	4	**21**	*4*

So, with no ties whatsoever, the first place goes to couple 22 and not to couple 21, which clearly would have been the winner by any common-sense-guided first look of the original marks.

The matter with this example is that, *when deciding about each dance, the traditional Skating system uses certain pieces of information which become lost when we go on to the* *dance summary*. For example, in Cha-cha-cha and Rumba couple 21 is first without discussion, while the second place of couple 22 in those dances is certainly not so clear-cut. In the other dances, however, both couples are closer to each other. However, since the dance summary according to the traditional Skating system keeps only the positions obtained in each dance, all such information is lost. The reader will agree that this fault has a much wider incidence than just the particular example above. As in § 2.2, it calls for replacing the positions obtained by a more quantitative measure of the degree of merit achieved.

In [**2**], this kind of anomalies were corrected by means of a variation that was called *Improved Skating System* (ISS). Its philosophy is exactly the same as the traditional Skating System. The only change is that after applying the Skating rules to each dance, the result is not just an ordering of the couples, but each of them is associated a certain fractional number. These fractional numbers are computed according to a formula that ensures that they rank the couples exactly in the same order as Rules 5-8. However, being fractional numbers, they contain much more information than just a placing. In fact, they somehow keep all the information used by Rules 5-8. So, if two couples had very similar marks, then their corresponding numbers are very close to each other. As in the traditional system, next step consists in adding these numbers over the different dances, just like Rule 9. More exactly, instead of an addition the Improved Skating System uses an average (arithmetic mean), which is of course equivalent, but in this context it has somehow a clearer meaning.

For instance, in the preceding example, the dance summary according to the Improved Skating System looks as follows, where A denotes the average (arithmetic mean) over dances.

Example B. ISS Dance Summary.

	Dances						
Nº	C	S	R	P	J	*A*	*P*
21	1.00000	1.00000	2.33333	2.33333	2.33333	**1.80000**	*1*
22	3.40448	3.40448	2.20250	2.20250	2.20250	**2.68329**	*2*
23	4.17500	4.96800	4.40000	5.00000	5.19000	**4.74660**	*5*
24	5.34667	5.94667	5.96000	5.95333	5.15000	**5.67133**	*6*
25	4.33493	3.96000	2.94667	2.94667	2.94667	**3.42699**	*3*
26	4.33333	4.36667	5.18000	4.20160	4.40320	**4.49696**	*4*

The main disadvantage of the Improved Skating System is that it does not lend itself to pen and paper implementation. This handicap is taken care of by the *Revised Skating System* (RSS) described in the following sections. As we shall see, it keeps the essentials of the Improved Skating System, and it has even better properties.

In § 7 we will discuss still another criticism against the Skating system. That criticism will motivate a further elaboration that will be called the *Double Revised Skating System* (DRSS). On the other hand, the DRSS will be based upon the RSS. This, together with the fact that the RSS makes sense by itself, has led us to proceed first with a description of the RSS.

3. A little terminology.

In order to make the procedure easier to describe we shall introduce first some terminology. Let us assume that we have a set of marks coming from different jugdes. In order to combine them, we shall make use of the following parameters.

A crucial role will be played by the **median** M, which is defined as follows: For an odd number of judges, it is the value that lies at the central position when the marks are arranged by order of magnitude; for an even number of judges, it is the average (arithmetic mean) of the two most central values in such an arrangement.

For example, the *median* of the numbers $(10, 9, 10, 5, 2, 5, 13)$ is the central value in $(2, 5, 5, \mathbf{9}, 10, 10, 13)$, namely 9. Similarly, the *median* of the numbers $(7, 6, 11, 10, 4, 8)$ is the arithmetic mean of the two most central values in $(4, 6, \mathbf{7}, \mathbf{8}, 10, 11)$, namely 7.5.

In the event of ties, we shall also make use of the **adjacent sums**, which are defined as follows: After the marks have been arranged by order of magnitude, the **1st adjacent sum** L_1 is the sum of the values lying at both sides of the median, for an odd number of judges, or at both sides of the two most central values, for an even number of judges; the **2nd adjacent sum** L_2 is the sum of the next two values at both sides of the ones already considered, and so on.

For example, the *2nd adjacent sum* of the numbers $(10, 9, 10, 5, 2, 5, 13)$ is the sum of the bold-faced values in $(2, \mathbf{5}, 5, 9, 10, \mathbf{10}, 13)$, namely 15, while the *1st adjacent sum* of the numbers $(7, 6, 11, 10, 4, 8)$ is the sum of the bold-faced values in $(4, \mathbf{6}, 7, 8, \mathbf{10}, 11)$, namely 16.

When dealing with different dances, the preceding parameters will be calculated for every dance (and every couple), and the results will be added up into the corresponding totals, which shall be denoted respectively by M^D, L_1^D, L_2^D, and so on.

4. The Revised Skating System.

This section is limited to a formal description of the procedure itself. After that, the following section will show its application to a couple of examples, and later on we shall discuss the performance of the method in comparison with the Traditional Skating System.

Starting point. For every dance of the final round, each of the judges ranks the couples from better to worse and marks them with the corresponding ordinal numbers, without any ties.

Step 1 (*Combining judges for each dance*). **1.1** : For every couple and every dance, the marks of the different judges are combined by calculating their *median* M.
1.2 : If separate dance results are required, they are obtained as follows: For every dance, couples are ranked according to the values of the *median* M, the lowest the better. In the event of ties, each group of tied couples is resolved by means of the *1st adjacent sum* L_1. If ties still persist, then the *2nd adjacent sum* L_2 shall be used, and so on.

Step 2 (*Combining dances*). **2.1** : For every couple, the medians obtained in the different dances are combined into their *sum total* M^D.
2.2 : The result of the competition is obtained by ranking the couples according to the values of this parameter, the lowest the better. In the event of ties, each group of tied couples is resolved by means of the *total 1st adjacent sum* L_1^D, i.e. the result of adding up the 1st adjacent sums corresponding to the different dances. If ties still persist, then the *total 2nd adjacent sum* L_2^D shall be used, and so on.

Let us remark that if all-round results are the only ones required, then Step 1.2 is not needed and adjacent sums need not be calculated unless Step 2.2 calls for it.

5. Examples.

The procedure is illustrated by means of the following examples, which will also serve for comparison with the TSS. Because of this last purpose, we have chosen cases where the results of the RSS are different from those of the TSS. Of course, in most cases both procedures will give the same results.

Example B. We begin by showing the application of the RSS to Example B of § 2.3. Step 1 is carried out in the following five tables, one for each dance. For completeness, in this case we calculate not only the all-round results, but also those for each particular dance.

Cha-cha-cha

	Judges													
Nº	A	B	C	D	E	*Rearranged*					M	L_1	L_2	P
21	1	1	1	1	1	1	1	1	1	1	**1**			*1*
22	6	6	3	3	3	3	3	3	6	6	**3**			*2*
23	4	4	2	5	4	2	4	4	4	5	**4**	8		*5*
24	3	3	6	6	5	3	3	5	6	6	**5**			*6*
25	5	2	4	2	6	2	2	4	5	6	**4**	7	8	*4*
26	2	5	5	4	2	2	2	4	5	5	**4**	7	7	*3*

Rumba

	Judges													
Nº	A	B	C	D	E	*Rearranged*					M	L_1	L_2	P
21	1	1	1	1	1	1	1	1	1	1	**1**			*1*
22	3	3	3	6	6	3	3	3	6	6	**3**			*2*
23	5	5	5	3	3	3	3	5	5	5	**5**			*5*
24	6	6	6	2	2	2	2	6	6	6	**6**			*6*
25	2	2	4	4	4	2	2	4	4	4	**4**	6		*3*
26	4	4	2	5	5	2	4	4	5	5	**4**	9		*4*

Samba

	Judges													
Nº	A	B	C	D	E	*Rearranged*					M	L_1	L_2	P
21	1	1	3	3	2	1	1	2	3	3	**2**	4	4	*1*
22	2	2	2	2	6	2	2	2	2	6	**2**	4	8	*2*
23	5	4	5	4	4	4	4	4	5	5	**4**			*4*
24	6	5	6	6	1	1	5	6	6	6	**6**			*6*
25	3	3	1	1	3	1	1	3	3	3	**3**			*3*
26	4	6	4	5	5	4	4	5	5	6	**5**			*5*

Paso Doble

	Judges													
Nº	A	B	C	D	E	*Rearranged*					M	L_1	L_2	P
21	1	1	3	3	2	1	1	2	3	3	**2**	4	4	*1*
22	2	2	2	2	6	2	2	2	2	6	**2**	4	8	*2*
23	5	5	5	5	5	5	5	5	5	5	**5**			*5*
24	6	6	6	4	1	1	4	6	6	6	**6**			*6*
25	3	3	1	1	3	1	1	3	3	3	**3**			*3*
26	4	4	4	6	4	4	4	4	4	6	**4**			*4*

Jive

	Judges													
Nº	A	B	C	D	E	*Rearranged*					*M*	L_1	L_2	*P*
21	1	1	3	3	2	1	1	2	3	3	**2**	4	4	*1*
22	2	2	2	2	6	2	2	2	2	6	**2**	4	8	*2*
23	5	5	6	4	5	4	5	5	5	6	**5**	10		*6*
24	6	4	5	5	1	1	4	5	5	6	**5**	9		*5*
25	3	3	1	1	3	1	1	3	3	3	**3**			*3*
26	4	6	4	6	4	4	4	4	6	6	**4**			*4*

The next table summarizes the medians obtained in the different dances and shows also their totals M^D. Notice that, in spite of the several ties that occurred within particular dances, the totals M^D do not contain any ties. Notice also that the final ranking coincides with the ISS (and therefore corrects the anomalous result of the TSS). In fact, if instead of the totals M^D we consider the averages $M^D/5$, their values are fairly similar to the averages A of the ISS dance summary.

RSS Dance Summary

	Dances (M)						
Nº	C	S	R	P	J	M^D	*P*
21	1	1	2	2	2	**8**	*1*
22	3	3	2	2	2	**12**	*2*
23	4	5	4	5	5	**23**	*5*
24	5	6	6	6	5	**28**	*6*
25	4	4	3	3	3	**17**	*3*
26	4	4	5	4	4	**21**	*4*

Example C. The next three-dance example illustrates the RSS in a case with ties in Step 2.1. In this case we calculate only the all-round results. Let us remark that, in practice, the 1st adjacent sums L_1 will not be calculated until it becomes necessary to break the tie that appears in the dance summary.

Waltz

	Judges																						
Nº	A	B	C	D	E	F	G	H	I	*Rearranged*									*M*	L_1	L_2	L_3	L_4
31	1	1	1	1	2	3	3	3	3	1	1	1	1	2	3	3	3	3	**2**				
32	2	2	2	6	6	2	2	2	6	2	2	2	2	2	2	6	6	6	**2**				
33	5	6	5	2	1	1	4	4	2	1	1	2	2	4	4	5	5	6	**4**	6			
34	3	3	3	3	3	6	1	1	1	1	1	1	3	3	3	3	3	6	**3**	6			
35	6	5	6	4	4	4	6	5	4	4	4	4	4	5	5	6	6	6	**5**				
36	4	4	4	5	5	5	5	6	5	4	4	4	5	5	5	5	5	6	**5**				

Tango

Nº	*Judges* A	B	C	D	E	F	G	H	I	*Rearranged*									*M*	L_1	L_2	L_3	L_4
31	2	2	2	2	2	3	3	3	3	2	2	2	2	2	3	3	3	3	**2**				
32	6	6	6	6	1	2	2	2	2	1	2	2	2	2	6	6	6	6	**2**				
33	1	1	1	5	3	1	1	1	5	1	1	1	1	1	1	3	5	5	**1**	2			
34	3	3	5	4	5	4	4	4	4	3	3	4	4	4	4	4	5	5	**4**	8			
35	4	4	3	1	4	5	5	5	1	1	1	3	4	4	4	5	5	5	**4**				
36	5	5	4	3	6	6	6	6	6	3	4	5	5	6	6	6	6	6	**6**				

Quickstep

Nº	*Judges* A	B	C	D	E	F	G	H	I	*Rearranged*									*M*	L_1	L_2	L_3	L_4
31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**1**				
32	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	**2**				
33	5	6	5	5	5	3	3	3	3	3	3	3	3	5	5	5	5	6	**5**	8			
34	3	3	3	3	3	6	6	6	6	3	3	3	3	3	6	6	6	6	**3**	9			
35	6	5	6	6	6	4	4	4	4	4	4	4	4	5	6	6	6	6	**5**				
36	4	4	4	4	4	5	5	5	5	4	4	4	4	4	5	5	5	5	**4**				

The final summary according to the RSS is shown next. In order to help comparing it with the TSS final summary, the latter is shown also at the right-hand side.

RSS Dance Summary

Nº	*Dances* (*M*) W	T	Q	M^D	L_1^D	L_2^D	L_3^D	L_4^D	*P*
31	2	2	1	**5**					*1*
32	2	2	2	**6**					*2*
33	4	1	5	**10**	16				*3*
34	3	4	3	**10**	23				*4*
35	5	4	5	**14**					*5*
36	5	6	4	**15**					*6*

TSS Dance Summary

Nº	*Dances* W	T	Q	*S*	*P*
31	2	3	1	**6**	*2*
32	1	2	2	**5**	*1*
33	4	1	6	**11**	*4*
34	3	4	3	**10**	*3*
35	6	5	5	**16**	*6*
36	5	6	4	**15**	*5*

6. Comparison with the Traditional Skating System.

In this section we discuss the performance of the Revised Skating System (RSS) in comparison with the Traditional Skating System (TSS). We shall consider first the case of an odd number of judges. The slightly trickier case of an even number of judges will be considered in § 6.6 below.

6.1. For an odd number of judges, *Step 1.1 of the RSS is exactly equivalent to Rules 5 and 8 of the TSS*, i. e. the rules that implement the fundamental principle of majority over judges.

In fact, the median is nothing else than the smallest value with the property that an absolute majority of the marks under consideration are less than or equal to that value. For instance, in the Cha cha cha of Example B, couple 24 has obtained a median of 5. This means that the better positions that this couple has been awarded by the different judges do not form an absolute majority (in this case 3 out of 5) unless we include up to the 5th position.

6.2. *Step 2 of the RSS is completely analogous to Rule 9 of the TSS.* In both cases, an addition over dances is carried out in order to comply with the second fundamental principle.

6.3. However, there is an important difference between both systems: After applying Step 1 of the RSS, or the equivalent Rules 5 and 8 of the TSS, couples can easily tie within particular dances. In that case, the TSS applies certain untying rules, namely Rules 6 and 7, while the RSS leaves these ties unresolved, at least for the moment (see §6.5). This is done completely on purpose, and it is one of the good features of the RSS.

In fact, *in its eagerness to break any ties immediately, the TSS easily falls into rather inequitable partial results, which later on can produce anomalies like those pointed out in* § 2.3. For instance, in the Cha cha cha of Example B, couples 25 and 26 have very similar marks, namely (5, 2, 4, 2, 6) and (2, 5, 5, 4, 2). In the TSS, this slight difference results in a 5th position (in Cha cha cha) for couple 25 against a 4th position for couple 26. In principle these results are just ordinal numbers, but, as we have already remarked, later on they acquire an absolute character as soon as we add them over the different dances. Certainly, to this effect it is much more equitable to give both couples a Cha cha cha global mark of 4 (the median). If we really have to decide which one of these couples was best in that dance, we probably will have to conclude that couple 25 was slightly in front of couple 26 (or maybe not, see § 12), but towards a global result it is much preferable to leave the tie unresolved. After all, such ties may disappear when combining dances.

6.4. *Using the median instead of ordinal numbers leads to more equitable results not only in the case of ties, as remarked above, but also in the absence of them.* For instance, in the Cha cha cha of Example B, couples 21 and 22 have respectively the following marks: (1, 1, 1, 1, 1) and (6, 6, 3, 3, 3). Both the TSS and the RSS agree that couple 21 is 1st and couple 22 is 2nd. However, instead of this last ordinal number, the RSS gives couple 22 a Cha cha cha global mark of 3 (the median). This is certainly more representative of their achievement, since not a single judge gave them a mark better than 3.

6.5. As we said above, the RSS does not worry about ties until it becomes necessary. Even then, the tie-resolving criteria used by the RSS can be considered more suitable than those of the TSS: In consonance with its fundamental use of the median, the RSS breaks the ties by looking at progressively extended adjacent sums. In so doing, the RSS systematically tries to satisfy the desirable property of being insensitive to the most extreme marks, both the highest and the lowest. In contrast, the TSS easily deviates from such a property: *in general the TSS tends to avoid using the highest marks, but the lowest ones may come into play a bit too early.*

6.6. In the less usual case of an even number of judges, the RSS usage of the median, which in this case is the average of the two most central marks, does not agree with the standard interpretation of the majority principle. For instance, let us consider the following example, where there were 6 couples but we are interested only in two of them:

Example D

	Judges				
Nº	A	B	C	D	*M*
41	1	1	6	6	**3.5**
42	5	5	5	5	**5**

By comparing the median values, the RSS considers couple 41 better than couple 42. In contrast, the TSS looks for an absolute majority in the standard sense, which puts couple 41 behind couple 42. Notice however that in this case an absolute majority in the standard sense is equivalent to a 75% qualified majority, while *the RSS interpretation through the use of the median is somehow closer to the ideal "more than 50%"*.

Summing up, both the TSS and the RSS comply with the fundamental principles stated in § 1, but the following features make the RSS more equitable than the TSS:

- Ties are not resolved until the end, thus limiting the effect of more or less arbitrary untying rules. Intermediate calculations are limited to an application of the fundamental principles.
- Using the median instead of simple ordinal numbers is more representative of the achievements of the different couples in each dance. In other words, the medians contain more information and therefore they are more suitable to be added over dances.
- When ties need to be resolved, this is done by certain criteria which do as much as possible to be insensitive to the most extreme marks, both the highest and the lowest.
- In the case of an even number of judges, the notion of absolute majority is not so far from the ideal "more than 50%".

7. New criticism: By-judge analysis and the Schiavo-Baricchi paradox.

Together with the Skating system, lately some people have been analyzing the results from a point of view which is referred to as "by-judge analysis" [**6**]. Like the traditional Skating system, this procedure has also two parts, but now the first part does not combine the different judges within each dance, but it combines the different dances for each judge, while the second part combines the different judges.

At first sight, this method looks as reasonable as the "by-dance analysis" of the traditional Skating system, as long as one complies with the same fundamental principles of giving all dances the same weight and combining judges by majority. However, in certain most important recent championships one has noticed that the by-judge analysis would lead to results which are in serious disagreement with those of the traditional Skating system.

Let us consider an example. Instead of any particular championship, for the moment we shall look at a simpler but essentially similar case with only 3 judges and 2 couples:

Example E. By-dance analysis

	WZ			TG			VW			SF			QS			*Dances*						
Nº	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	W	T	V	F	Q	*S*	*P*
51	1	1	2	1	2	1	2	1	1	2	2	2	2	2	2	1	1	1	2	2	**7**	*1*
52	2	2	1	2	1	2	1	2	2	1	1	1	1	1	1	2	2	2	1	1	**8**	*2*

Example E. By-judge analysis

	A					B					C					*Judges*				
Nº	W	T	V	F	Q	W	T	V	F	Q	W	T	V	F	Q	A	B	C	*M*	*P*
51	1	1	2	2	2	1	2	1	2	2	2	1	1	2	2	8	8	8	**8**	*2*
52	2	2	1	1	1	2	1	2	1	1	1	2	2	1	1	7	7	7	**7**	*1*

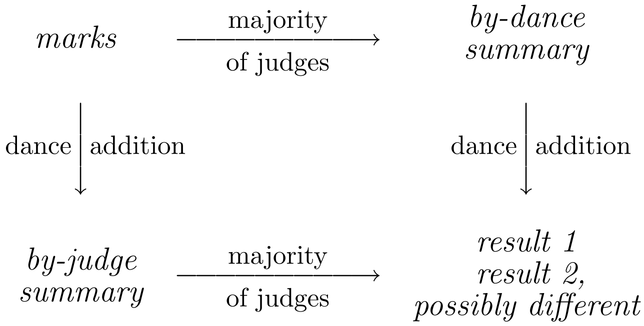
The first table shows the marks grouped by dances and the dance summary according to the traditional Skating system. Couple **51** wins by a sum of 7 against the 8 of couple **52**.

The second table shows the same marks grouped by judge, together with the judge summary. In compliance with Principle 2, the judge summary has been obtained by adding the marks of each judge in the different dances. Later on, we will discuss the general suitability of such a method, but in the present case all reasonable methods lead to the same result: for each judge the overall winner is couple 52, and therefore this common result is also the result of the by-judge analysis.

So, we have two similar points of view that lead to contradictory results.

Of course, with more couples things can become a little more involved, but the paradox remains equally possible.

The next diagram illustrates the essentials of the paradox. Starting from the upper left corner, the traditional by-dance analysis consists in moving first to the right and then downwards, while the by-judge analysis consists in moving first downwards and then to the right. In dancing, both paths would take you to the same point, but in scrutineering they can lead to different results!



To quote a real case, the by-judge analysis of the *2002 United Kingdom Open Amateur Latin Championship* (*Dance News*, 1754, p. 5) reveals that *a majority of 7 judges out of 11 preferred the couple that the traditional Skating system had relegated to second position!*

More recently, the very same two couples have been involved again in such a paradox in the *2002 IDSF European Latin Championship*.

Similar things have happened before in the following cases (at least): the *2001 United Kingdom Open Professional Standard Championship*; the *2000 World Professional Standard Championship*; the *1994 German Open Professional Standard Championship*. In these three cases the couples involved were Augusto Schiavo & Caterina Arzenton and Luca & Lorraine Baricchi. Because of this, this phenomenon is sometimes called the "Schiavo-Baricchi paradox".

8. Questions posed by the preceding paradox.

Since such paradoxical cases have occurred repeatedly in recent important championships, one is naturally led to pose the following questions [**6**]:

- Question 1. How often can such paradoxical cases happen?
- Question 2. Can the Skating system be modified so as to avoid such a paradox?
- Question 3. Is there any good reason to prefer the by-dance analysis to the by-judge analysis?
- Question 4. Can the Skating system be modified so as to combine both points of view?

In the following we give an answer to each of these questions.

Answer to Question 1: First of all, it is clear that such paradoxical cases will happen only when two couples are close enough to each other. Of course, one could begin to search in past competition records, but this takes some time.

Alternatively, one can use a computer to automatically generate millions of possibilities where the occurrence of the paradox is checked. Instead of generating those cases completely at random, it is reasonable to concentrate on certain conditions where the paradox is more likely to happen. Following this idea, we have considered the following conditions: an odd number of judges; 5 dances; the first and second placings are monopolized by two couples, of which one wins three dances and the other wins the two remaining dances (notice that Rules 10 and 11 will not come into play). Given the number of adjudicators, we have looked at the cases that satisfy the conditions just mentioned, and we have simply counted how many of them were paradoxical (when the majority principle is understood in terms of medians). The results are displayed in the following table (where four-figure percentages are based on an exhaustive inspection of all possible cases, while the three-figure ones are based on a random sample of ten million cases):

Number of judges:	3	5	7	9	11	13	15	17	19
Percentage of paradoxical cases:	26.95	30.65	31.93	32.6	32.9	33.2	33.4	33.5	33.6

Although they concern certainly a rather specific situation, these results suggest that *the Schiavo-Baricchi paradox occurs probably more often than expected.*

Answer to Question 2: Unfortunately, it looks like the problem is not easily avoidable. More specifically, *for any procedure that complies with the fundamental principle of combining judges by majority there will always be cases where the by-judge analysis will not agree with the by-dance analysis.*

In fact, as the reader will easily convince himself, cases like example E above will always be paradoxical, no matter which procedure we use as soon as judges are combined by majority. More generally, the same will happen with all of the paradoxical cases that we have seen to occur so often when the first and second placings are monopolized by two couples. In particular, the RSS is no exception to this rule.

Having said that, when the positions involved are not only the first and second, then some procedures may prove more successful than others in avoiding the paradox. In this connection, the RSS looks certainly better than the TSS. For instance, in examples B and C the TSS produces in certain sense a total of five ocurrences of the paradox, while the RSS reduces it to only once.

Answer to Question 3: Since there is no way to completely avoid the Schiavo-Baricchi paradox, the question arises of whether there is any good reason to prefer the by-dance analysis to the by-judge analysis or viceversa.

In favour of the by-dance analysis as we have been doing it until now, one can argue that, after all, couples compete to win dances, not to “win adjudicators”. In fact, the Syllabus of the Blackpool Dance Festival reads for example as follows: *The British Professional Modern Ballroom Dancing Championship ... Prizes will be awarded to the First, Second, Third, Fourth, Fifth and Sixth in each dance. The Couple showing the best ‘All-round’ standard in the four dances will be declared the British Professional Modern Champions.* If we take that statement and we replace the word ‘dance’ by ‘judge’ it does not make much sense.

On the other hand, the by-judge analysis is also a very natural point of view. In fact, it is the main motivation of the popular *How the adjudicators saw it* section of *Dance News*. In support of this point of view, one could argue that the main purpose of a dancesport competition is not so much to rank the couples in each particular dance, but to find out which couples are the best ones in the whole set of dances included. If this is true, then one should certainly pay attention to what the judges are expressing in that connection.

Of course, judges do not directly assess the all-round efficiency of each couple, but after each dance they express their particular preferences in that dance by means of an ordering of the couples. Therefore, the information that they provide about each couple is a collection of ordinal numbers corresponding to the different dances. Certainly, from those numbers one should be able to derive some information about the all-round preferences of the judges.

Answer to Question 4: If one wants to take into account both points of view, by-dance and by-judge, then a natural thing to do is to balance their results by some sort of average.

The remainder of this paper presents a specific method of this kind that we call "Double Revised Skating System" (DRSS).

9. The Double Revised Skating System.

The starting point and Steps 1 and 2 of the DRSS are the same as for the RSS but without Steps 1.2 and 2.2. After that, the DRSS proceeds to a by-judge analysis and a final balance:

Starting point. For every dance of the final round, each of the judges ranks the couples from better to worse and marks them with the corresponding ordinal numbers, without any ties.

Step 1 (*Combining judges for each dance*). For every couple and every dance, the marks of the different judges are combined by calculating their *median* M.

Step 2 (*Combining dances*). For every couple, the medians obtained in the different dances are combined into their *sum total* M^D.

Step 3 (*Combining dances for every judge*). For every couple and every judge, the marks corresponding to the different dances are combined by calculating their *sum*.

Step 4 (*Combining judges*). For every couple, the results obtained in Step 3 for the different judges are combined by calculating their *median* M^J.

Step 5 (*Balance*). **5.1**: For every couple, the results obtained in Steps 2 and 4 are combined by calculating their *sum* $M^B = M^D + M^J$. The global result is obtained by ranking of the couples according to the values of this parameter, the lowest the better.
5.2: In the event of ties, then each group of tied couples is resolved according to the results of an analogous procedure with the median being replaced by the *1st adjacent sum*. If ties still persist, then the *2nd adjacent sum* shall be used and so on.

10. Examples.

In the following, the results of Step 2 (dance summary) are indicated by means of a superscript ^{D}, those of Step 4 (judge summary) by means of a superscript ^{J}, and those of Step 5 (final balance) by means of a superscript ^{B}.

Example B. The next table shows the results of Steps 2–5. The first part reproduces the dance summary obtained in § 5. The next part contains the judge summary: for each judge there is one column containing the results of Step 3, i. e. the sum of the marks that he has awarded to each couple over all dances. These results are the input for Step 4, which is carried out next by means of a rearrangement in order to find out their median M^J (median of sums). Finally, the last part shows the results of Step 5, i. e. the value of $M^B = M^D + M^J$, and the resulting global positions P.

Summary

Nº	*Dances* C	S	R	P	J	M^D	*Judges* A	B	C	D	E	*Rearranged*					M^J	M^B	*P*
21	1	1	2	2	2	**8**	5	5	11	11	8	5	5	8	11	11	**8**	**16**	*1*
22	3	3	2	2	2	**12**	15	15	12	15	27	12	15	15	15	27	**15**	**27**	*2*
23	4	5	4	5	5	**23**	24	23	23	21	21	21	21	23	23	24	**23**	**46**	*5*
24	5	6	6	6	5	**28**	27	24	29	23	10	10	23	24	27	29	**24**	**52**	*6*
25	4	4	3	3	3	**17**	16	13	11	9	19	9	11	13	16	19	**13**	**30**	*3*
26	4	4	5	4	4	**21**	18	25	19	26	20	18	19	20	25	26	**20**	**41**	*4*

Notice that in the preceding table couple 21 are the winners both by dances and by judges. So, in contrast with the TSS, the RSS avoids the Schiavo-Baricchi paradox in connection with couple 21 . On the other hand, couples 22 and 25 are still "in paradox": 22 is in front of 25 in the dance summary, while the judge summary orders them the other way round. By balancing both points of view, the RSS has finally concluded in favour of couple 22 .

Alternative way of arranging calculations. In pen and paper practice, adding marks over different dances, as required for the judge summary, is a bit of an inconvenience. In this connection, it may help to arrange calculations in a different way, where marks are tabulated by couples, instead of dances. Each of these tables contains one column for each judge and one row for each dance. These data are then combined in two ways: horizontally, to calculate medians, and vertically, to calculate sums.

Next tables show this alternative arrangement in the case of Example B. Although empty, we have included the adjacent sum columns in order to show how to proceed in the general case.

21	*Judges* A	B	C	D	E	*Rearranged*					*M*	L_1	L_2
C	1	1	1	1	1	1	1	1	1	1	**1**		
S	1	1	1	1	1	1	1	1	1	1	**1**		
R	1	1	3	3	2	1	1	2	3	3	**2**		
P	1	1	3	3	2	1	1	2	3	3	**2**		
J	1	1	3	3	2	1	1	2	3	3	**2**		
D								·			**8**		
J	5	5	11	11	8	5	5	8	11	11	**8**		
B											**16**		

22	*Judges* A	B	C	D	E	*Rearranged*					*M*	L_1	L_2
C	6	6	3	3	3	3	3	3	6	6	**3**		
S	3	3	3	6	6	3	3	3	6	6	**3**		
R	2	2	2	2	6	2	2	2	2	6	**2**		
P	2	2	2	2	6	2	2	2	2	6	**2**		
J	2	2	2	2	6	2	2	2	2	6	**2**		
D								·			**12**		
J	15	15	12	15	27	12	15	15	15	27	**15**		
B											**27**		

23	*Judges*												
	A	B	C	D	E			*Rearranged*			M	L_1	L_2
C	4	4	2	5	4	2	4	4	4	5	**4**		
S	5	5	5	3	3	3	3	5	5	5	**5**		
R	5	4	5	4	4	4	4	4	5	5	**4**		
P	5	5	5	5	5	5	5	5	5	5	**5**		
J	5	5	6	4	5	4	5	5	5	6	**5**		
D								·			**23**		
J	24	23	23	21	21	21	21	23	23	24	**23**		
B											**46**		

24	*Judges*												
	A	B	C	D	E			*Rearranged*			M	L_1	L_2
C	3	3	6	6	5	3	3	5	6	6	**5**		
S	6	6	6	2	2	2	2	6	6	6	**6**		
R	6	5	6	6	1	1	5	6	6	6	**6**		
P	6	6	6	4	1	1	4	6	6	6	**6**		
J	6	4	5	5	1	1	4	5	5	6	**5**		
D								·			**28**		
J	27	24	29	23	10	10	23	24	27	29	**24**		
B											**52**		

25	*Judges*												
	A	B	C	D	E			*Rearranged*			M	L_1	L_2
C	5	2	4	2	6	2	2	4	5	6	**4**		
S	2	2	4	4	4	2	2	4	4	4	**4**		
R	3	3	1	1	3	1	1	3	3	3	**3**		
P	3	3	1	1	3	1	1	3	3	3	**3**		
J	3	3	1	1	3	1	1	3	3	3	**3**		
D								·			**17**		
J	16	13	11	9	19	9	11	13	16	19	**13**		
B											**30**		

26	*Judges*												
	A	B	C	D	E			*Rearranged*			M	L_1	L_2
C	2	5	5	4	2	2	2	4	5	5	**4**		
S	4	4	2	5	5	2	4	4	5	5	**4**		
R	4	6	4	5	5	4	4	5	5	6	**5**		
P	4	4	4	6	4	4	4	4	4	6	**4**		
J	4	6	4	6	4	4	4	4	6	6	**4**		
D								·			**21**		
J	18	25	19	26	20	18	19	20	25	26	**20**		
B											**41**		

Summary

	Dances			*Judges*			*Balance*			
Nº	M^D	L_1^D	L_2^D	M^J	L_1^J	L_2^J	M^B	L_1^B	L_2^B	P
21	**8**			**8**			**16**			*1*
22	**12**			**15**			**27**			*2*
23	**23**			**23**			**46**			*5*
24	**28**			**24**			**52**			*6*
25	**17**			**13**			**30**			*3*
26	**21**			**20**			**41**			*4*

Example F. The next example comes from a real championship of those mentioned in § 6 where the Schiavo-Barichi paradox involved the winners and runners-up. In this case, the global balance results in a tie, which must be resolved by means of the *1st adjacent sum*.

Cha-cha-cha

	Judges											*Rearranged*																
Nº	A	B	C	D	E	F	G	H	I	J	K												M	L_1	L_2	L_3	L_4	L_5
4	4	1	1	2	2	2	1	2	2	2	4	1	1	1	2	2	2	2	2	2	4	4	**2**	4				
36	6	6	6	5	5	4	5	6	5	6	3	3	4	5	5	5	5	6	6	6	6	6	**5**					
67	3	3	3	3	4	5	4	4	3	4	5	3	3	3	3	3	4	4	4	4	5	5	**4**					
135	2	4	5	6	3	3	3	3	4	3	2	2	2	3	3	3	3	3	4	4	5	6	**3**					
152	5	5	4	4	6	6	6	5	6	5	6	4	4	5	5	5	5	6	6	6	6	6	**5**					
202	1	2	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	**1**	2				

Samba

	Judges											*Rearranged*																
Nº	A	B	C	D	E	F	G	H	I	J	K												M	L_1	L_2	L_3	L_4	L_5
4	2	1	1	1	1	1	2	1	3	1	5	1	1	1	1	1	1	1	2	2	3	5	**1**	2				
36	6	6	6	5	5	6	5	6	5	6	3	3	5	5	5	5	6	6	6	6	6	6	**6**					
67	4	3	3	3	4	4	3	5	2	4	4	2	3	3	3	3	4	4	4	4	4	5	**4**					
135	3	4	5	6	3	5	4	3	4	3	2	2	3	3	3	3	4	4	4	5	5	6	**4**					
152	5	5	4	4	6	3	6	4	6	5	6	3	4	4	4	5	5	5	6	6	6	6	**5**					
202	1	2	2	2	2	2	1	2	1	2	1	1	1	1	1	2	2	2	2	2	2	2	**2**	4				

Rumba

	Judges											*Rearranged*																
Nº	A	B	C	D	E	F	G	H	I	J	K												M	L_1	L_2	L_3	L_4	L_5
4	1	2	1	1	2	2	2	2	3	2	3	1	1	1	2	2	2	2	2	2	3	3	**2**	4				
36	6	5	6	4	5	6	5	6	6	5	4	4	4	5	5	5	5	6	6	6	6	6	**5**					
67	3	4	4	3	6	3	3	5	2	4	5	2	3	3	3	3	4	4	4	5	5	6	**4**					
135	4	3	5	5	3	4	4	3	4	3	2	2	3	3	3	3	4	4	4	4	5	5	**4**					
152	5	6	3	6	4	5	6	4	5	6	6	3	4	4	5	5	5	6	6	6	6	6	**5**					
202	2	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	**1**	2				

Paso Doble

Nº	*Judges* A	B	C	D	E	F	G	H	I	J	K	*Rearranged*											*M*	L_1	L_2	L_3	L_4	L_5
4	2	1	1	1	1	1	1	3	3	2	1	1	1	1	1	1	1	1	2	2	3	3	**1**	2				
36	6	5	5	4	4	6	5	6	5	6	3	3	4	4	5	5	5	5	6	6	6	6	**5**					
67	4	3	4	3	5	3	4	5	4	3	5	3	3	3	3	4	4	4	4	5	5	5	**4**					
135	3	4	6	6	3	5	3	2	2	4	4	2	2	3	3	3	4	4	4	5	6	6	**4**					
152	5	6	3	5	6	4	6	4	6	5	6	3	4	4	5	5	5	6	6	6	6	6	**5**					
202	1	2	2	2	2	2	2	1	1	1	2	1	1	1	1	2	2	2	2	2	2	2	**2**	4				

Jive

Nº	*Judges* A	B	C	D	E	F	G	H	I	J	K	*Rearranged*											*M*	L_1	L_2	L_3	L_4	L_5
4	2	1	1	1	1	2	2	1	3	1	4	1	1	1	1	1	1	2	2	2	3	4	**1**	3				
36	5	5	5	5	5	5	4	6	5	6	2	2	4	5	5	5	5	5	5	5	6	6	**5**					
67	3	4	3	3	4	3	5	5	4	4	5	3	3	3	3	4	4	4	4	5	5	5	**4**					
135	4	6	6	6	3	4	3	4	2	3	3	2	3	3	3	3	4	4	4	6	6	6	**4**					
152	6	2	4	4	6	6	6	3	6	5	6	2	3	4	4	5	6	6	6	6	6	6	**6**					
202	1	3	2	2	2	1	1	2	1	2	1	1	1	1	1	1	2	2	2	2	2	3	**2**	3				

Dance Summary

Nº	*Dances (M)* C	S	R	P	J	M^D	*Dances (L_1)* C	S	R	P	J	L_1^D
4	2	1	2	1	1	**7**	4	2	4	2	3	15
36	5	6	5	5	5	**26**						
67	4	4	4	4	4	**20**						
135	3	4	4	4	4	**19**						
152	5	5	5	5	6	**26**						
202	1	2	1	2	2	**8**	2	4	2	4	3	15

Judge Summary

Nº	*Judges* A	B	C	D	E	F	G	H	I	J	K	*Rearranged*											M^J	L_1^J
4	11	6	5	6	7	8	8	9	14	8	17	5	6	6	7	8	8	8	9	11	14	17	**8**	16
36	29	27	28	23	24	27	24	30	26	29	15	15	23	24	24	26	27	27	28	29	29	30	**27**	
67	17	17	17	15	23	18	19	24	15	19	24	15	15	17	17	17	18	19	19	23	24	24	**18**	
135	16	21	27	29	15	21	17	15	16	16	13	13	15	15	16	16	16	17	21	27	29	21	**16**	
152	26	24	18	23	28	24	30	20	29	26	30	18	20	23	24	24	26	26	28	29	30	30	**26**	
202	6	10	10	9	8	7	7	7	5	7	6	5	6	6	7	7	7	7	8	9	10	10	**7**	14

Final Balance

Nº	M^B	L_1^B	P
4	**15**	31	*2*
36	**53**		*6*
67	**38**		*4*
135	**35**		*3*
152	**52**		*5*
202	**15**	29	*1*

11. Concluding remarks.

In this paper we have presented two alternative versions of the Skating system, namely the *Revised Skating System* (RSS) and the *Double Revised Skating System* (DRSS). As we have seen, they correct certain bad behaviour of the *Traditional Skating System* (TSS).

As we remarked in § 6, the RSS has better properties than the TSS even if we restrict ourselves to a by-dance analysis, or to a one dance competition. On the other hand, the DBSS is an elaboration of the RSS that takes care of the Schiavo-Baricchi paradox. Both methods have the virtue of lending themselves easily to pen and paper calculation.

To make it easier to test them, we have made available a little computer application, called *MiniSkate* [**7**], which implements the RSS and the DRSS and allows comparing their results with those of the TSS.

In spite of their good properties, however, the RSS and the DRSS can still be criticized for two main reasons:

First of all, although the situation is not so bad as in the TSS, the RSS amd the DRSS are still liable to the "Viennese Waltz paradox" described in § 2.2. Even assuming the same performances and the same preferences by the judges, the winner may depend on how many other couples are present. The weakest point in this connection is Step 3 of the DRSS, where we are still adding ordinal numbers.

On the other hand, one can argue that the Schiavo-Baricchi paradox is not really removed, but only taken care of rather artificially. The trick of averaging the results of the by-dance analysis and the by-judge analysis is rather gratuitous. Why do we give both approaches the same weight? Why don't we simply accept the results of the by-judge analysis?

Such questions will be considered in more detail in [**8**], where the problem will be analyzed from a broader perspective. There, we expect to benefit from recent developments in social choice theory, an active branch of social science that deals with voting processes of all kinds, and where the analogous of both the Viennese Waltz paradox and the Schiavo-Baricchi paradox have been known for some time. It seems likely that such an exploration will lead to other alternative systems worthy of consideration.

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