THE SKATING SYSTEM

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2nd edition. July 2001[1]

The Skating system is the standard procedure for determining the results of dancesport competitions. As it is well-known, it has a certain degree of complexity. Unavoidably, the present exposition is influenced by the fact that I am a mathematician, and I hope it benefits from that. On the one hand, this influence has led me to reformulate the traditional procedure in a systematic way, which hopefully will contribute to a better understanding of it. On the other hand, the mathematical point of view has led me to think of a variant that in my opinion is an improvement upon the traditional system.

The following section aims at giving an overview accessible to anybody. In contrast, Sections 2–5 ask for a minimal affinity with the mathematical style there used. My proposal of an improved Skating system does not appear until Sections 6 and 7.

1. Introduction.

A dancesport competition aims at determining the **couples** that better execute a certain set of **dances**, as well as ordering these couples according to their efficiency. This is done by means of a certain number of **adjudicators**.

If the number of couples that take part is above a certain value, then the competition includes one or more **rounds** before proceeding with the **final**. The rounds before the final are used to carry out a gradual selection of the participant couples until reaching a certain number of finalists. Usually one aims at having six of them.

In every non-final round, the aim is to **select** a certain number of couples for the next round. In contrast, in the final the aim is to **order** the couples according to their efficiency. In so doing, one generally needs to **combine** the opinions of several different adjudicators, as well as the results of several dances.

In the non-final rounds this combining is done in a very simple way. For every dance, each adjudicator makes his own selection of couples. From these data, one computes for each couple the total number of marks obtained from the different adjudicators in the different dances. The couples are then ranked according to decreasing values of this number of marks, and finally one selects those that lead this ranking until a certain position which is decided by the competition authority, always respecting the principle that tied couples must receive the same treatment.

In the final, the problem of combining the ratings coming from different adjudicators and different dances is more involved. For every dance, each adjudicator ranks the couples from better to worse according to his opinion. The problem is to suitably combine these multiple partial rankings so as to obtain a final global one. The usual way to do it is the so-called Skating system.

This system consists of two parts. In the first part one deals with each dance separately and the aim is to combine the ratings given for that dance by the different adjudicators. This results in a combined rating of the couples for each dance. In the second part, these combined

[1] The first edition of this paper appeared in catalan on October 1993.

ratings corresponding to the different dances are combined between them so as to obtain the final global result.

In both parts we are concerned with combining several orderings into one. A simple and natural way to do that is to use the **average**, or, equivalently, the sum of the ordinal numbers corresponding to the same couple in the different orderings that are to be combined. Next table shows that in an example where there are 6 couples and 3 orderings to be combined. Each row corresponds to a different couple, columns A, B, C contain the three input orderings, S gives the sum of the ordinal numbers coming from these three orderings, X gives the corresponding average, and L gives the resulting combined ordering:

	Data					
Nº	A	B	C	S	X	L
11	3	5	3	11	$3\frac{2}{3}$	**4**
12	1	1	5	7	$2\frac{1}{3}$	**2**
13	2	2	2	6	2	**1**
14	4	4	1	9	3	**3**
15	5	6	6	17	$5\frac{2}{3}$	**6**
16	6	3	4	13	$4\frac{1}{3}$	**5**

This method is certainly very reasonable when the orderings that we are combining belong to different dances. In fact, in some sense the smallness of S is a measure of the overall efficiency of the couple in those dances.

However, when the orderings that we are combining come from different adjudicators then the averaging method is more disputable. In fact, assume that in the preceding example we were combining the orderings given by three adjudicators in the same dance. As one can see, couple 12 would have obtained the second position in spite that an absolute majority of adjudicators considered that they should have been the winners. This is indeed very questionable.

This is the reason why, when combining the orderings given by different adjudicators, the Skating system uses another method where what matters is the opinion of an absolute **majority** of adjudicators.

In order to give a first idea of how does this method work, perhaps the best is to see it applied to a particular case, like for instance the one of next table:

	Judges					*Computations*					
Nº	A	B	C	D	E	@1	@2	@3	@4	@5	L
21	6	5	5	1	2	-	-	-	-	4	**6**
22	2	2	2	4	1	-	4				**1**
23	1	6	1	2	6	-	3				**2**
24	5	4	4	5	5	-	-	-	-	5	**5**
25	3	1	6	6	3	-	-	$3^{(7)}$			**3**
26	4	3	3	3	4	-	-	$3^{(9)}$			**4**

In this case, there is no couple with an absolute majority of first places. In such a situation, one considers not only the first places but also the second ones, and one seeks whether both possibilities together amount to an absolute majority of adjudicators. In our example, this happens with two couples, namely numbers 22 and 23. But couple 22 has 4 marks of that kind, while couple 23 has only 3. Owing to this fact, couple 22 gets the first position, and couple 23 becomes second. In order to allocate the subsequent positions among the remaining couples, one looks now for those couples that have obtained an absolute majority of marks ranging from first to third place. Among the remaining couples there are two that satisfy that condition, namely couples 25 and 26. This time both of them have 3 marks ranging from first to third place. But couple 25 is considered better than couple 26 because the sum of these three marks is lower (although, owing to the other two marks, which put couple 25 in the last place, the sum of the marks of all adjudicators is larger than that of couple 26). With this, couple 25 gets the third position and couple 26 becomes fourth. Following the same pattern, now one examines the remaining couples looking for those that have obtained an absolute majority of marks ranging from first to fourth place. But, since none of them satisfies this condition, one goes on and considers the marks ranging from first to fifth place. Both remaining couples, 21 and 24, have an absolute majority of marks in that range. But couple 21 has only 4, while couple 24 has 5, and therefore the last one is considered better (although the sum of all marks is larger than that of couple 21). With this, couple 24 obtains therefore the fifth position and couple 21 finishes sixth. The information used along this procedure is summarized in the *Computations* section of the preceding table. Later on we will explain precisely the notation used there.

Once this procedure has been applied to every dance, the Skating system proceeds to combine the results of the different dances by applying now the average method. As we have already explained, this method amounts to ordering the couples according to the sum S of the combined results obtained in the different dances. As we have said above, the smallness of this parameter is a measure of the overall efficiency of the couple in the set of dances included in the competition.

Of course, it may happen, and it is not rare at all, that several couples obtain the same value of S. In this case the Skating system resolves the ties by applying certain rules, somewhat involved, which are known as Rules 10 y 11. Later on we will deal with them in detail.

The majority criterion entered the ballroom dancing scene in 1938, when it was used in the *Star Championships* (England). The name *"Skating"* by which the present system is known indicates that this criterion must have originated from figure skating (which nowadays uses a completely different system). When it started to be used in ballroom dancing competitions, the majority criterion did not have its present fundamental character. It was used in the particular case that a couple had been placed first by an absolute majority of adjudicators, but in any other case the winner was determined by means of the average criterion. The majority criterion did not reach its present scope until the end of the nineteen forties, when the *Official Board of Ballroom Dancing* (the present *British Dance Council*), adopted the present method in order to combine the orderings given by different adjudicators. The present form of the Skating system was completed in 1956, when that board introduced the last tie-resolving rule (Rule 11) in combining the results corresponding to different dances. At present, the Skating system is very established over the world, and in particular it is the procedure prescribed by the *International Dance Sport Federation (IDSF)*.

The best reputed exposition of the Skating system in its present form is the booklet of Arthur Dawson (1963) that we list in the bibliography [**1**]. Together with that work, one must mention also an anonymous abridged version of it [**2**]. These expositions follow a case approach that includes most of the cases that can occur in practice, but some extreme cases are not explicitly dealt with so that it is not clear how to work them out. For instance, such is the case of Example 4 of the present work.

With this state of affairs, there was a need for a more systematic exposition that covered all possible cases, no matter how little probable they could be. An attempt at such a systematization can be found in the handbook *Grundlagen der Turnierleitung* edited by *Deutscher Tanzsportverband (DTV)*, from Germany [**4**]. Unfortunately, however, the results there do not agree with those of [**1**]. Such a disagreement happens for instance in Example N of [**1**, p. 14–15], and in Example 15 of [**4**, Anhang, S. 18–19, 22].

In what follows we give a formulation that covers absolutely all possible cases and agrees with [**1**] in all cases there considered. This is achieved in a systematic and succint way by means of a precise notation and terminology, much in the style of mathematics.

After the first edition of the present work, there have appeared several new expositions of the Skating system, which we list in the Bibliography. In particular, [**6**] and [**9**] stand out as traditional expositions that succeed in covering all possible cases.

2. Terminology.

To begin with, we must precise the meaning of several terms of which me will make an extensive usage.

At every moment we will be considering a certain **group** of couples among which one must allocate a series of consecutive **positions**. These positions will be indicated by their corresponding ordinal numbers. The best, that is the lowest in number, of the positions to be allocated among the group will be denoted by m. Similarly, the highest in number of the positions to be allocated will be denoted by M. The interval of consecutive positions associated with the group will be expressed by means of a dash joining both numbers. For instance, the interval 4–6 refers to a group of three couples tied for the positions 4 to 6. In general, the number of couples in the group is given by $M - m + 1$. At the beginning, the group under consideration consists of all finalist couples, m is equal to 1, and M is equal to the number of finalist couples.

The way to allocate positions within a group will be to **order** its member couples according to the values of a certain **parameter** x. If better positions should correspond to larger values of x, we will say that we are ordering the couples according to the **criterion** "x large"; in the contrary case we will say that we are ordering them according to the criterion "x small".

If there are several couples with the same value of x, then we will say that these couples are **tied** with respect to that parameter. These couples will make up a new group, with m equal to the lowest in number of the positions involved in the tie. In order to resolve the tie, this new group will have to be dealt with by means of a new criterion. When we say that a group must be dealt with by means of a certain series of **successive criteria**, it means that if the first criterion gives rise to ties, then each group of tied couples must be dealt with by means of the second criterion, and so on.

Sometimes, a criterion, or a series of successive criteria, will not be used for ordering all couples of a group, but only for extracting the **leading couple or couples**, that is, the best couple of the group according to that criterion or series of criteria, or, in case of a tie, the group of the best couples.

3. Combining the orderings given by different adjudicators.

As we said above, the Skating system begins by considering each dance separately with the aim of combining the ratings given for that dance by the different adjudicators. This is done on the basis of the following parameters of every couple in that dance:

j_n : number of adjudicators that have placed the couple in position n,

k_n : number of adjudicators that have placed the couple in position n or better,

s_n : sum of the positions better or equal to n obtained from the different adjudicators,

r : first value of n for which k_n represents an absolute majority of adjudicators.

Thus, for instance, in the second table of the Introduction couple 22 has $r = 2$, $k_2 = 4$, $j_2 = 3$, and $s_2 = 7$.

From the information provided by these parameters, the combined ordering is obtained by means of the following procedure:

Rule 1–8: The participant couples are ordered according to the following successive criteria:

"r small", "k_r large", "s_r small", "k_{r+1} large", "k_{r+2} large", and so on.

If after applying all these criteria some tie still exists, then each of the tied couples is assigned the average of the positions involved in the tie.

As one can easily see, the preceding list of successive criteria is equivalent to the following one:

"r small", "k_r large", "s_r small", "j_{r+1} large", "j_{r+2} large", and so on.

Another equivalent way of specifying the list of successive criteria of Rule 1–8 is:

"k_1^* large", "s_1^* small", "k_2^* large", "s_2^* small", and so on,

where k_n^* and s_n^* represent the following parameters:

k_n^* : if k_n represents an absolute majority of adjudicators, $k_n^* = k_n$; otherwise, $k_n^* = 0$,

s_n^* : if k_n represents an absolute majority of adjudicators, $s_n^* = s_n$; otherwise, $s_n^* = 0$.

4. Combining the results corresponding to different dances.

Once the preceding procedure has been applied to each of the dances included in the competition, then the Skating system proceeds to combine the results of the different dances. This is done on the basis of the following parameters of every couple:

S : sum of the positions obtained in the different dances,

l_n : number of dances where the couple has obtained position n or better,

t_n : sum of the positions better or equal to n obtained in the different dances.

k_n, s_n, k_n^*, s_n^* : parameters analogous to those of Section 3 but determined from the detail of places given by the different adjudicators over all dances; in particular, here an absolute majority means a number strictly larger than one half of the product of the number of adjudicators by the number of dances.

From this information, the final global ordering is obtained by means of the following procedure (where we recall that m denotes the lowest in number of the positions to be allocated among a group):

Rule 9: The participant couples are ordered according to the criterion "S small".
In case of ties, each group of tied couples is dealt with by means of Rule 10.

Rule 10: From each group one extracts the leading couple or couples according to the successive criteria "l_m large" and "t_m small".

Having done that, the remainder of the group is dealt with in the same way (with m increased by the number of extracted couples), and so on.

If, at any moment of applying this rule, the leading couples are more than one, then these couples are considered tied by this rule, and each group of tied couples is dealt with by means of Rule 11.

Rule 11: From each group one extracts the leading couple or couples according to the following successive criteria:

"k^*_m large", "s^*_m small", "k^*_{m+1} large", "s^*_{m+1} small", and so on.

Having done that, the remainder of the group is dealt with by means of Rule 10 (with m increased by the number of extracted couples).

If when applying Rule 11, the leading couples are more than one, then these couples are considered tied and each of them is assigned the average of the positions involved in the tie.

It is worth remarking that in the definition of the parameters l_n and t_n one does not require an absolute majority (since we are combining different dances and not different adjudicators). Furthermore, in Rule 10 the parameters l_n and t_n are examined only for $n = m$, i. e. one does not begin from $n = 1$ (unless m is equal to 1), nor does one proceed towards values of n larger than m.

In contrast, in Rule 11 the parameters k^*_n and s^*_n require an absolute majority (over the product of adjudicators and dances). On the other hand, in Rule 11 the parameters k^*_n and s^*_n are examined for $n = m$ and, if necessary, also for larger values of n, i. e., as in Rule 10, one does not begin from $n = 1$ (unless m is equal to 1), but here one does proceed towards values of n larger than m.

Finally, let us stress that Rules 10 and 11 are not used to order all couples of a tied group, but only to extract the leading ones. In both cases, if the remaining couples are more than one, they are again dealt with by means of Rule 10. Next figure illustrates the interplay of Rules 9, 10 and 11.

$$\begin{tikzcd}
 & & & & \textit{best} \arrow[r, "\text{tie}"] & \text{average} \\
 & & \textit{best} \arrow[r, "\text{tie}"] & \boxed{\textit{R11}} \arrow[ur] \arrow[dr] & & \\
\boxed{\textit{R9}} \arrow[r, "\text{ties}"] & \boxed{\textit{R10}} \arrow[ur] \arrow[dr] & & & \textit{remaining} \arrow[r, "\text{several}"] & \boxed{\textit{R10}} \\
 & & \textit{remaining} \arrow[r, "\text{several}"] & \boxed{\textit{R10}} & &
\end{tikzcd}$$

5. Examples.

In the following we refer to the values of k_n and s_n by means of the notation $@n$. The value of s_n will be made explicit only when needed, and in that case it will be indicated next to the value of k_n between parentheses, in a smaller size and slightly risen. So, $@4 = 5^{(13)}$ means that $k_4 = 5$ and $s_4 = 13$. We will do likewise for the values of l_n and t_n. A hyphen in the place corresponding to $@n$ means that we are dealing with k_n and s_n and the value of k_n does not constitute a majority.

Example 3. This example comes from a real competition (where the author formed part of couple 6).

Waltz

	Judges							*Computations*						
Nº	A	B	C	D	E	F	G	@1	@2	@3	@4	@5	@6	*L*
2	4	3	4	3	4	2	6	-	-	-	6			**4**
6	3	7	1	1	1	7	7	-	-	4				**3**
24	2	2	6	5	7	1	2	-	$4^{(7)}$	4	4	5	6	**2**
30	5	6	2	2	2	6	1	-	$4^{(7)}$	4	4	5	7	**1**
53	1	5	5	7	5	3	5	-	-	-	-	6		**7**
71	6	4	3	6	3	5	3	-	-	-	$4^{(13)}$	5		**5**
77	7	1	7	4	6	4	4	-	-	-	$4^{(13)}$	4		**6**

Tango

	Judges							*Computations*						
Nº	A	B	C	D	E	F	G	@1	@2	@3	@4	@5	@6	*L*
2	2	3	5	2	4	2	6	-	-	$4^{(9)}$				**3**
6	3	7	1	1	3	6	7	-	-	$4^{(8)}$				**2**
24	1	2	6	5	7	1	2	-	4					**1**
30	6	5	2	4	1	7	1	-	-	-	4			**5**
53	4	6	3	7	5	3	5	-	-	-	-	5		**6**
71	7	4	4	6	6	5	3	-	-	-	-	4		**7**
77	5	1	7	3	2	4	4	-	-	-	5			**4**

Foxtrot

	Judges							*Computations*						
Nº	A	B	C	D	E	F	G	@1	@2	@3	@4	@5	@6	*L*
2	2	3	5	2	6	2	5	-	-	$4^{(9)}$				**2**
6	4	7	1	1	2	6	7	-	-	-	$4^{(8)}$			**4**
24	1	2	7	4	7	1	3	-	-	$4^{(7)}$				**1**
30	5	5	4	5	1	7	1	-	-	-	-	6		**7**
53	3	6	2	7	5	4	4	-	-	-	$4^{(13)}$	5	6	**6**
71	6	4	3	6	4	5	2	-	-	-	$4^{(13)}$	5	7	**5**
77	7	1	6	3	3	3	6	-	-	$4^{(10)}$				**3**

Final Summary

	Dances				*Computations*			
Nº	V	T	F	*S*	R9	R10	R11	*L*
2	4	3	2	9	2–3	2–3 ($@2=1^{(2)}$)	3	**3**
6	3	2	4	9	2–3	2–3 ($@2=1^{(2)}$)	2	**2**
24	2	1	1	4	1			**1**
30	1	5	7	13	4–5	5 ($@4=1$)		**5**
53	7	6	6	19	7			**7**
71	5	7	5	17	6			**6**
77	6	4	3	13	4–5	4 ($@4=2$)		**4**

Aplication of Rule 11

Nº	*Waltz* A	B	C	D	E	F	G	*Tango* A	B	C	D	E	F	G	*Foxtrot* A	B	C	D	E	F	G	*Computations* @2	@3	*L*
2	4	3	4	3	4	2	6	2	3	5	2	4	2	6	2	3	5	2	6	2	5	-	$11^{(26)}$	**3**
6	3	7	1	1	1	7	7	3	7	1	1	3	6	7	4	7	1	1	2	6	7	-	$11^{(18)}$	**2**

Example 4. The following example has been made up in order to illustrate the application of the procedure in an extreme case.

Cha-cha-cha

Nº	*Judges* A	B	C	D	E	*Computations* @1	@2	@3	@4	@5	@6	*L*
41	5	5	4	6	5	-	-	-	-	4		**5**
42	4	4	6	5	4	-	-	-	3			**4**
43	7	6	3	2	2	-	-	$3^{(7)}$	3			**3**
44	1	1	1	1	1	5						**1**
45	3	2	7	7	7	-	-	-	-	-	-	**7**
46	2	3	2	4	6	-	-	$3^{(7)}$	4			**2**
47	6	7	5	3	3	-	-	-	-	3		**6**

Rumba

Nº	*Judges* A	B	C	D	E	*Computations* @1	@2	@3	@4	@5	@6	*L*
41	5	6	6	4	7	-	-	-	-	-	4	**6**
42	4	5	4	6	4	-	-	-	3			**4**
43	2	2	2	1	1	-	5					**1**
44	1	1	5	5	5	-	-	-	-	5		**5**
45	3	4	7	7	6	-	-	-	-	-	3	**7**
46	7	3	1	2	2	-	3					**2**
47	6	7	3	3	3	-	-	3				**3**

Samba

Nº	*Judges* A	B	C	D	E	*Computations* @1	@2	@3	@4	@5	@6	*L*
41	3	5	4	2	3	-	-	3				**3**
42	5	7	7	6	6	-	-	-	-	-	3	**7**
43	2	2	5	7	7	-	-	-	-	$3^{(9)}$		**5**
44	1	1	6	4	4	-	-	-	4			**4**
45	7	3	2	3	2	-	-	4				**2**
46	6	6	3	5	5	-	-	-	-	$3^{(13)}$		**6**
47	4	4	1	1	1	3						**1**

Paso Doble

Nº	Judges A	B	C	D	E	Computations @1	@2	@3	@4	@5	@6	L
41	5	5	1	1	1	3						**1**
42	4	6	4	4	4	-	-	-	4			**4**
43	2	2	5	7	7	-	-	-	-	3		**5**
44	1	1	3	5	5	-	-	3				**3**
45	6	3	2	2	3	-	-	4				**2**
46	7	7	6	3	2	-	-	-	-	-	3	**7**
47	3	4	7	6	6	-	-	-	-	-	4	**6**

Jive

Nº	Judges A	B	C	D	E	Computations @1	@2	@3	@4	@5	@6	L
41	5	5	5	5	4	-	-	-	-	5		**5**
42	4	4	2	1	1	-	$3^{(4)}$					**1**
43	2	2	6	6	7	-	-	-	-	-	4	**6**
44	1	1	7	7	6	-	-	-	-	-	3	**7**
45	6	6	1	2	2	-	$3^{(5)}$					**2**
46	7	7	3	3	3	-	-	3				**3**
47	3	3	4	4	5	-	-	-	4			**4**

Final Summary

Nº	Dances C	R	S	P	J	S	Computations R9	R10	R10	R11	R10	R11	R10	R11	L
41	5	6	3	1	5	20	1–7	1–5 ($@1=1$)		2–5	2–5 ($@2=1^{(1)}$)	3–5	3–4 ($@3=2^{(4)}$)	4	**4**
42	4	4	7	4	1	20	1–7	1–5 ($@1=1$)		2–5	2–5 ($@2=1^{(1)}$)	3–5	5 ($@3=1$)		**5**
43	3	1	5	5	6	20	1–7	1–5 ($@1=1$)		2–5	2–5 ($@2=1^{(1)}$)	2			**2**
44	1	5	4	3	7	20	1–7	1–5 ($@1=1$)		1					**1**
45	7	7	2	2	2	20	1–7	6–7 ($@1=0$)	7 ($@6=3$)						**7**
46	2	2	6	7	3	20	1–7	6–7 ($@1=0$)	6 ($@6=4$)						**6**
47	6	3	1	6	4	20	1–7	1–5 ($@1=1$)		2–5	2–5 ($@2=1^{(1)}$)	3–5	3–4 ($@3=2^{(4)}$)	3	**3**

Aplication of Rule 11

Nº	Cha-cha-cha A	B	C	D	E	Rumba A	B	C	D	E	Samba A	B	C	D	E	Paso Doble A	B	C	D	E	Jive A	B	C	D	E	Computations @1	@2	@3	@4	@5	@6
41	5	5	4	6	5	5	6	6	4	7	3	5	4	2	3	5	5	1	1	1	5	5	5	5	4	-	-	-	-	21	24
42	4	4	6	5	4	4	5	4	6	4	5	7	7	6	6	4	6	4	4	4	4	4	2	1	1	-	-	-	15	18	23
43	7	6	3	2	2	2	2	2	1	1	2	2	5	7	7	2	2	5	7	7	2	2	6	6	7	-	13	14	14	16	19
44	1	1	1	1	1	1	1	5	5	5	1	1	6	4	4	1	1	3	5	5	1	1	7	7	6	13	13	14	16	21	23
45	3	2	7	7	7	3	4	7	7	6	7	3	2	3	2	6	3	2	2	3	6	6	1	2	2	-	-	14	15	15	19
46	2	3	2	4	6	7	3	1	2	2	6	6	3	5	5	7	7	6	3	2	7	7	3	3	3	-	-	13	14	16	20
47	6	7	5	3	3	6	7	3	3	3	4	4	1	1	1	3	4	7	6	6	3	3	4	4	5	-	-	-	16	18	22

The preceding example shows clearly how in general the process of combining the results of different dances consists in a successive partitioning of the couples into gradually smaller groups, to end up, if possible, with isolated couples.

6. Criticism and variants.

6.1. Some additional examples. Sometimes, the results of the Skating system can be rather surprising. Thus, in Example 5 shown next the winner is couple 52 (thanks to Rule 10), but in the detail of places given by the different adjudicators over all dances one easily notices that this couple has received only 9 first places, while couple 51 has received 16 first places and yet obtains the second overall position.

Example 5

	Cha-cha-cha					*Rumba*					*Samba*					*Paso Doble*					*Jive*					*Dances*					
Nº	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	C	R	S	P	J	*L*
51	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	1	1	2	2	2	1	1	2	2	2	1	1	2	2	2	**2**
52	6	6	3	3	3	6	6	2	2	2	6	6	1	1	1	6	6	1	1	1	6	6	1	1	1	3	2	1	1	1	**1**
53	3	3	6	6	6	2	2	6	6	6	2	2	6	6	6	2	2	6	6	6	2	2	6	6	6	6	6	6	6	6	**6**
54	4	2	4	5	4	4	3	4	5	4	4	3	3	3	5	4	4	4	5	4	4	4	4	3	4	4	4	3	4	4	**4**
55	5	5	2	4	5	5	4	5	4	5	5	5	5	5	4	5	3	5	4	5	5	3	5	4	5	5	5	5	5	5	**5**
56	2	4	5	2	2	3	5	3	3	3	3	4	4	4	3	3	5	3	3	3	3	5	3	5	3	2	3	4	3	3	**3**

Even more surprising is Example 6, where, in spite of not having obtained any first place, the winner is couple 62, while couple 61 has 16 first places and yet obtains the second overall position (without the need of Rules 10 and 11).

Example 6

	Cha-cha-cha					*Rumba*					*Samba*					*Paso Doble*					*Jive*					*Dances*					
Nº	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	C	R	S	P	J	*L*
61	1	1	1	1	1	1	1	1	1	1	1	1	3	3	2	1	1	3	3	2	1	1	3	3	2	1	1	2	2	2	**2**
62	6	3	3	3	3	2	3	3	6	6	2	2	2	2	6	2	2	2	2	6	2	2	2	2	6	2	2	1	1	1	**1**
63	4	4	2	5	4	5	5	5	3	3	5	4	5	4	4	5	5	5	5	5	5	5	6	4	5	3	5	4	5	6	**5**
64	3	5	6	6	6	6	6	6	2	2	6	5	6	6	1	6	6	6	4	1	6	4	5	5	1	6	6	6	6	5	**6**
65	5	2	4	2	5	3	2	4	4	4	3	3	1	1	3	3	3	1	1	3	3	3	1	1	3	4	3	3	3	3	**3**
66	2	6	5	4	2	4	4	2	5	5	4	6	4	5	5	4	4	4	6	4	4	6	4	6	4	5	4	5	4	4	**4**

In spite that such cases can occur relatively often, the Skating system is rarely a subject of criticism. At any rate, the possibility of such cases is considered a minor imperfection compared with the virtue of obeying to the **principle of the majority of adjudicators**, that is, for each dance the result is obtained according to the opinion of an absolute majority of adjudicators.

6.2. Simplified Skating system (*Majoritätssystem*). Even so, some people have remarked that Rules 10 and 11 do not fit in with another principle, also very desirable, which we shall call **principle of the equivalence of dances**, that is, when combining the results of the different dances all of them should have exactly the same weight. Because of that, some dancesport regulations give up untying the couples that obtain the same value of S, or alternatively they do it by means of an additional round between the tied couples. In the tie is not resolved then the tied couples are assigned the average of the positions at stake. Such a procedure has a place for instance in the regulations of the *Deutcher Tanzsportverband (DTV)*, from Germany, where it receives the name of *Majoritätssystem* [**3**]. Of course, its main drawback is precisely the possibility of ties.

6.3. Improved Skating system. Really, the only way to comply with the principle of the equivalence of dances is to average (or to add up) the values of analogous parameters corresponding to each dance. In the traditional Skating system the rating of each dance is reduced to the obtained ordinal number, which makes it relatively easy that its average over all dances leads to ties. One way to avoid this is to replace that ordinal number by a more sensitive parameter whose value arranges the couples in the same order as Rule 1–8. Although at first sight it might not be clear how to choose such a parameter, certainly one often sees that two couples are closer or farther than what is shown by the ordinal number resulting from Rule 1–8. For instance, that is the case of couples 24 and 30 of Example 3 in Waltz. Therefore, what we are looking for is a parameter Y that quantifies the different criteria used by Rule 1–8, and combines them in such a way that those rules are exactly equivalent to ordering the couples according to the values of that parameter.

In the following we point out a way of choosing such a parameter Y. Unfortunately, it is given by a rather complicated algebraic formula. This is an important drawback when the scrutiny is done by hand, but it does not involve any special difficulty when it is programmed on a computer, as it is nowadays often the case.

The formula for Y is the following:

$$Y = r \;+\; \left(1 - \frac{k_r}{J}\right) \;-\; \frac{1}{J}\left(1 - \frac{s_r}{r\,k_r}\right) \;+\; \frac{2}{J^3 N}\left((J - k_r) - \sum_{n>r} j_n\,(2/J)^{n-r-1}\right),$$

where j_n, k_n, s_n, r are the parameters introduced in Section 3, while J and N stand for the following parameters:

J : number of adjudicators,

N : number of couples in the final.

This formula starts from the value of r and continues with a series of corrections that successively take into account the effects of k_r, s_r, j_{r+1}, j_{r+2}, and so on. These corrections are chosen in such a way that each of them acts in the direction required by Rule 1–8, and that its effect can never be counteracted by the subsequent terms. As a consequence, the result of ordering the couples according to increasing values of Y is exactly equivalent to Rule 1–8, and one can give a mathematical proof that this is always the case. Typically, the first correction has an order of magnitude of tenths, the second of hundredths and the third of thousandths or even smaller.

If we agree that the parameter Y gives a more precise rating of the different couples in each dance, then we must accept as reasonable that the global rating of each couple in the set of dances be based upon the parameter

Z : average of the values of Y obtained in the different dances.

In other words, instead of Rules 9, 10, and 11, we are proposing to use simply the following:

Rule 9bis: The participant couples are ordered according to the criterion “Z small”.

The following tables show the application of this procedure to Examples 3–6. For every dance there is a column which shows the value of Y obtained by each couple in that dance; the following columns show the average Z of these values and the resulting global position L.

Example 3

Nº	*Dances* W	T	F	Z	L
2	4.11964	3.39422	3.39481	3.63622	**2**
6	3.35958	3.38334	4.35927	3.70073	**3**
24	2.41312	2.39526	3.37067	2.72635	**1**
30	2.41311	4.35850	5.10059	3.95740	**4**
53	5.11488	5.25774	4.40315	4.92525	**7**
71	4.40298	5.40059	4.40298	4.73552	**6**
77	4.40391	4.24362	3.40711	4.01821	**5**

Example 4

Nº	*Dances* C	R	S	P	J	Z	L
41	5.19000	6.17500	3.37915	1.40428	4.99200	4.22809	**4**
42	4.40137	4.40137	6.38889	4.20137	2.33608	4.34582	**5**
43	3.35961	1.96000	5.32274	5.32274	6.13333	4.41969	**7**
44	1.00000	4.93600	4.12637	3.31385	6.28889	3.93302	**1**
45	6.94857	6.34444	3.16881	3.16859	2.37095	4.40027	**6**
46	3.35748	2.36889	5.37333	6.32222	3.40428	4.16524	**3**
47	5.34804	3.40406	1.40384	6.15833	4.17500	4.09785	**2**

Example 5

Nº	*Dances* C	R	S	P	J	Z	L
51	1.00000	1.00000	1.96000	1.96000	1.96000	1.57600	**1**
52	3.40448	2.40499	1.40520	1.40520	1.40520	2.00501	**2**
53	5.96000	5.94667	5.94667	5.94667	5.94667	5.94933	**6**
54	4.17500	4.18750	3.40160	4.20000	3.99000	3.99082	**4**
55	4.96800	4.98400	4.99200	4.97600	4.97600	4.97920	**5**
56	2.40384	3.20160	3.98000	3.20160	3.40320	3.23805	**3**

Example 6

Nº	*Dances* C	R	S	P	J	Z	L
61	1.00000	1.00000	2.33333	2.33333	2.33333	1.80000	**1**
62	3.20224	3.38226	2.20250	2.20250	2.20250	2.63840	**2**
63	4.17500	4.96800	4.40000	5.00000	5.19000	4.74660	**5**
64	5.97333	5.94667	5.96000	5.95333	5.15000	5.79667	**6**
65	4.33333	3.97000	2.94667	2.94667	2.94667	3.42867	**3**
66	4.33493	4.36667	5.18000	4.20160	4.40320	4.49728	**4**

By comparing these tables with the corresponding ones of Sections 5 and 6.1, the reader can check that for every dance the values Y determine the same order as Rule 1–8. One can notice that in the first dance of Example 3 couples 24 and 30 obtain very close values of Y, which obeys to the fact that both couples differ only in the values of j_6 and j_7. In contrast, the traditional method does not reflect the fact that this difference has very little significance indeed. One can also notice that in the first dance of Example 4 couple 44 obtains the exact value $Z = 1.0000$ since all adjudicators agreed in putting that couple in the first place.

In most of the cases that happen in practice, the final order obtained by this procedure coincides with the result of the traditional system. However, in certain especial cases, like are indeed the preceding examples, these two methods give different final results. The positive part of it is that, owing to this fact, one is able to get rid of the undesired phenomena that, as we have seen in Section 6.1, can happen with the traditional Skating system. On the other hand, one should not forget that both procedures do always agree in each separate dance, and that the final results can differ only in couples whose detailed marks have similar merit.

Finally, one will agree that ties under Rule 9bis will be extremely improbable (like in the case of using Rules 10 and 11). On the other hand, they can never be ruled out completely, and if they happen there are no other possibilities than running an additional round between the tied couples or drawing lots (or not resolving the tie).

7. Conclusion.

Summing up, we can distinguish the following three variants of the Skating system:

- Traditional Skating System: Rules 1–8, 9, 10, and 11.
- Simplified Skating System (*Majoritätssystem*): Rules 1–8, 9.
- Improved Skating System: Rules 1–8, 9bis.

In the separate dance stage, i. e. when dealing with each dance separately in order to combine the orderings given for it by the different adjudicators, all of these three variants respect the principle of the majority of adjudicators, and the resulting ordering for each dance is exactly the same. So, the differences lie in the dance combining stage, i. e. when combining the results corresponding to the different dances.

In the traditional system, Rules 10 and 11 make the probability of ties virtually nil, but this is achieved at the expense of introducing certain criteria that are alien to the dance equivalence principle, i. e. the principle that in combining the different dances all of them should have exactly the same weight. On the other hand, as we have seen in Section 6.1, sometimes the results of the traditional system can diverge from common sense.

The simplified system, known in Germany as *Majoritätssystem*, does without Rules 10 y 11, and so it scrupulously respects the dance equivalence principle. In exchange, it has the disadvantage of having to admit frequent ties. On the other hand, as it is shown by Example 6, its results can still diverge from common sense.

Finally, the improved system is the one that we have introduced in the preceding section. This system respects also scrupulously the dance equivalence principle, but at the same time the probability of ties is virtually nil and it avoids the possibility of results diverging from common sense like those seen above.

If any, the main disadvantage of the improved system is that it does not lend itself to manual computation. In contrast, its implementation in a computer program presents no problems and in fact is much easier that in the case of the traditional system. This is due mainly to the absence of the intricate Rules 10 and 11. On the other hand, the use of the parameter Y simplifies also the implementation of Rule 1–8, which in fact is useful for the programming of the other variants as well.

Finally, let us remark that the quantitative character of the improved system might be of interest to strengthen the recognition of dancesport as a full-fledged sport.

8. Bibliography.

[**1**] Arthur Dawson, 1963.
The Skating system. Working out the marks in ballroom dancing championships.
Published by the *Official Board of Ballroom Dancing* (the present *British Dance Council, BDC*).

[**2**] (Original author unknown).
Abridged exposition of the Skating system based upon [**1**].
Reproduced in:
- *International Council of Amateur Dancers* (the present *International DanceSport Federation, IDSF*), 1981: *Skating System.*
- Richard M. Moroney, 1994. *Description of the skating system used to judge ballroom dance events.*

Available in:
- `http://laurent.riesterer.free.fr/skating/rules.html`.
- `http://www.bath.ac.uk/ su3bds/bcbd/skating.html`.
- `http://www.centralhome.com/ballroomcountry/how_ballroom_competitions_judged.htm`.

[**3**] *Deutscher Tanzsportverband (DTV)*, 2001.
Das Majoritätssystem, in *Turnier- und Sportordnung des Deutschen Tanzsportverbandes e. V. (TSO). Stand: Januar 2001.* (Anhang, S. 68–71).
Available in: `http://www.topturnier.de/tso/tso_anh2.htm`.

[**4**] *Deutscher Tanzsportverband (DTV)*, 1991.
Grundlagen der Turnierleitung. 2 Auflage.
Tanzwelt Verlag GmbH. (Anhang, S. 10–24).

[**5**] Xavier Mora, 1993. *El sistema Skating.*
First edition of the present work.
Published by the *Asociación Española de Baile Deportivo y de Competición, AEBDC.*

[**6**] Jeff Carlsen, 1996.
Certified Correct. The dancesport scrutineer's rulebook and observer's guide to competition marks.
Dancing Bear Publishing Company, Halifax, Canadá.
Endorsed by the *National Dance Council of America (NDCA).*

[**7**] Ferran Rovira, 1997.
¿Porqué ganamos, porqué perdemos?
TopDance, 15, 16.
Available in: `http://inicia.es/de/ballrun/skating.htm`

[**8**] Ralf Pickelmann, 1999.
Das Skatingsystem.
Available in: `http://www.tbw.de/rpcs/skating/`.

[**9**] Jim Warren, 2000.
The mystical art of tabulating dancesport competition marks (The Skating System).
Available in: `http://www.csd.co.za/articles/scrutineer/Intro.htm`.

[**10**] Robet J. Steele, (date unknown).
Interpretation and Application of the Skating System.
See `http://www.fatd.com.au/bookandvideo.html`.

[**11**] Estelle Grassby, 2001.
The A to Z of Scrutineering.
Approved by the *British Dance Council (BDC).*

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